Technion - Israel Institute of Technology

Mechanical Engineering Faculty

Control Systems Engineering

ME Project

Optimization of Telepresence in Bilateral Teleoperation

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Contents

1 Abstract

First of all, let's explain the domain of "bilateral teleoperation": the prefix "tele" means "in remote", so, "teleoperation" means "remotely operating". The notion of teleoperation refers to the human capability to move objects by remote operation.
Usually, this teleoperation is performed by a joystick at the operator's side - this sub system is called the master - and this joystick's motion commands to the remotely operated system - this sub system is called the slave. In a general setting, the operator applies a force on the master joystick which results in a displacement that is transmitted to the slave sub system that mimics that same movement. When the master sub system is also controlled by a controller which receives feedback signals from the slave sub system, the teleoperator is said to be controlled bilaterally.
Historical time line of the bilateral teleoperation domain researches can be found in "Bilateral teleoperation: An historical survey", by Peter F. Hokayem and Mark W. Spong.
Many researches in this domain included consideration of the delay which is always present because of communication means between the master and the slave (in both directions). These delays might influence the stability of the system and can even lead to instability.
The main goals for the designer of a bilateral teleoperated system are:
- maintain stability of the system,
and
- optimize the coupling between the master and the salve sub systems. This is sometimes called the "telepresence".

Most of the research works that have been performed were passivity based methods, which ensured Robust Stability of the system, despite the above mentioned delays. These methods were not intuitive for the telepresence optimizing. There were in the past research works on Optimization of the telepresence - for example "Teleoperation controller design using H^∞ optimization with application to motion-scaling", by J. Yan and S. Salcudean. Nevertheless, the Optimization research work has not been developed enough, due to the problems caused by the delays. The consequence was that the solutions found were not satisfying of not easy to work with. In the last years, efforts were provided in order to find solutions for the problem of bilateral teleoperation with delays - for example "Decentralized H^2 optimal control of delayed bilateral teleoperation systems" by Maxim Kristalny and Jang Ho Cho. This reopened the subject of finding criteria for optimization

of the telepresence. Therefore, the question of which criteria should be used for the telepresence optimization is to be answered. In this document, the delays were not considered, in order to simplify the model, and its aim is to find and compare criteria for the telepresence optimization.

In bilateral teleoperation, each sub system (master and slave) can be controlled by its own Controller - the system is then said to be controlled by decentralized control. If both sub systems are controlled by the same controller, the system is said to be controlled by centralized control.

In this document, a case study of bilateral teleoperation is exposed, controlled by centralized control, in which the whole system is controlled by the same unique controller. The system includes the master and the slave sub systems and both of them are modelled by a simple and same joystick which can be manipulated by a simple DC motor.

The modelled system does not consider the delays which would obviously exist in a real system.

The aim of this document is to check criterions for coupling of the master and the slave optimization, or in other terms, check criterions for telepresence optimization - while always maintaining the stability of the system.

The theory used in this document has been developed in "Decentralized H^2 optimal control of delayed bilateral teleoperation systems" by Maxim Kristalny and Jang Ho Cho.

2 Modelling Part

2.1 Master System Modelling

The Joystick is modelled by a simple aluminium beam, while its length (L) and the radius (r) of its section have been chosen arbitrary. The angular stiffness (kT) and angular damping (cT) at the bottom of the Joystick have been taken into consideration.
The angular moment of inertia (J) is then calculated, using also the density of aluminium(see Figure 1).
Assuming that in the master's side, the Operator's hand is modelled by a simple mass-spring-damper system $[m_1, k_1, c_1]$. The coordinates of the system are x and θ while:
- x is the horizontal position of the "hand system" m_1 - assuming oscillations of the joystick small enough, no vertical motion of this system is taken into account,
- θ is the angular inclination of the joystick.

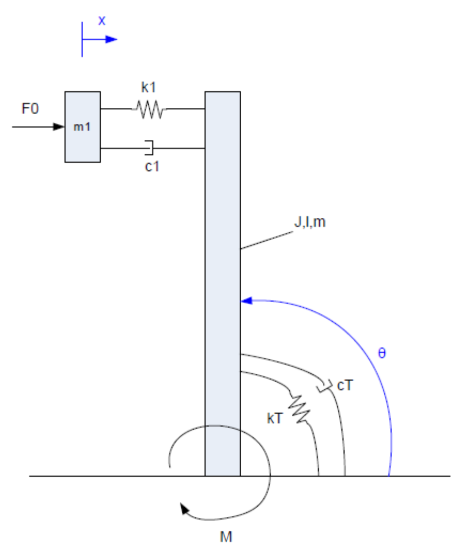


Figure 1: Model of the Master system

Parameters definition and values:
Joystick parameters:
L = 0.1 [m] , r = 0.015 [m]
kT = 0.48 [N/rad] , cT = 0.1 [N.sec/rad] ; (values explained later on)

$\rho = 2700$ [kg/m^3], M $= \rho * L * \pi * r^2$
J $= 1/4 * M * r^2 + 1/3 * M * L^2$
Master's hand parameters:
$m_1 = 1$ [kg], $c_1 = 3$ [N.sec/m], $k_1 = 200$ [N/m]

Let's find the equations of motion of the Master system, depicted in Figure 1.
The two states of the Master system are (θ, x).

$$\begin{cases} J_0\ddot{\theta} + c_T\dot{\theta} + k_T\theta = M + l(c_1(\dot{x} - l\cos\theta\dot{\theta}) + k_1(x - l\sin\theta)) \\ m_1\ddot{x} + c_1(\dot{x} - l\cos\theta\dot{\theta}) + k_1(x - l\sin\theta) = F_0 \end{cases} \tag{1}$$

Linearizing by using the fact that theta will stay small enough in order to assume:

$$cos(\theta) \approx 1$$
$$sin(\theta) \approx \theta$$

We get:

$$\begin{cases} J_0\ddot{\theta} + c_T\dot{\theta} + k_T\theta = M + l(c_1(\dot{x} - l\dot{\theta}) + k_1(x - l\theta)) \\ m_1\ddot{x} + c_1(\dot{x} - l\dot{\theta}) + k_1(x - l\theta) = F_0 \end{cases} \tag{2}$$

Performing the Laplace transformation (with initial conditions equal to zero):

$$\begin{cases} (J_0s^2 + c_Ts + k_T)\theta(s) = M + l(c_1s + k_1)x(s) - l^2(c_1s + k_1)\theta(s) \\ (m_1s^2 + c_1s + k_1)x(s) - l(c_1s + k_1)\theta(s) = F_0 \end{cases} \tag{3}$$

After few algebraic calculus, we get:
$\theta = \theta(F_0, M)$:

$$\begin{aligned} \theta(s) &= \frac{l(c_1s + k_1)}{(J_0s^2 + c_Ts + k_T)(m_1s^2 + c_1s + k_1) + l^2m_1(c_1s + k_1)s^2}F_0(s) \\ &+ \frac{m_1s^2 + c_1s + k_1}{(J_0s^2 + c_Ts + k_T)(m_1s^2 + c_1s + k_1) + l^2m_1(c_1s + k_1)s^2}M(s) \end{aligned} \tag{4}$$

and $x = x(F_0, M)$:

$$\begin{aligned} x(s) &= \frac{(J_0s^2 + c_Ts + k_T)(m_1s^2 + c_1s + k_1) + l^2m_1(c_1s + k_1)s^2 + l^2(c_1s + k_1)^2}{(m_1s^2 + c_1s + k_1)[(J_0s^2 + c_Ts + k_T)(m_1s^2 + c_1s + k_1) + l^2m_1(c_1s + k_1)s^2]}F_0(s) \\ &+ \frac{l(c_1s + k_1)}{(J_0s^2 + c_Ts + k_T)(m_1s^2 + c_1s + k_1) + l^2m_1(c_1s + k_1)s^2}M(s) \end{aligned} \tag{5}$$

Let's assume that the master's and the slave's joysticks are driven by DC motors.

Parameters of the DC motor:
Km = 72.6e-3; motor's torque (armature) constant
Kb = 24.2e-3; motor's back electromotive force (emf) constant
R = 2.21; motor's windings resistance
N = 1; transmission between motor's shaft to Joystick angular velocities

The Closed Loop from M - the motor's Torque - to V - the input Voltage - is:

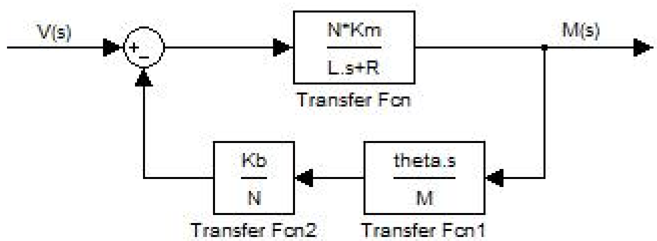


Figure 2: M/V closed loop

Then, assuming that L≈0, the resulting transfer function is:

$$\frac{M}{V} = \frac{K_m N[(J_0 s^2 + c_T s + k_T)(m_1 s^2 + c_1 s + k_1) + l^2 m_1 (c_1 s + k_1) s^2]}{[(J_0 s^2 + c_T s + k_T)(m_1 s^2 + c_1 s + k_1) + l^2 m_1 (c_1 s + k_1) s^2] R + K_m K_b s (m_1 s^2 + c_1 s + k_1)} \quad (6)$$

Therefore, considering results (4) and (6), we get:
$\theta = \theta(F_0, V):$

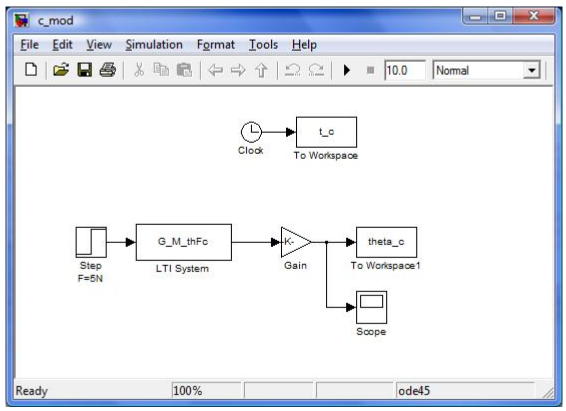
$$\theta(s) = \frac{\theta(s)}{F_0(s)} F_0 + \frac{\theta(s)}{M(s)} \frac{M(s)}{V(s)} V$$

Determination of kT:
The aim is to get a maximum angle of the joystick of 60°when the input Force of the Hand of the Operator has been taken as equal to 5 N. For this simulation: cT = 0. Then, in steady state, kT must verify:

$$\begin{aligned} kT\theta_{max} &= 5l \\ kT &= 0.48 \end{aligned}$$

Determination of the value cT has been done using simulink.

The aim is to get a behavior of the joystick near to normal behavior of a real joystick: with few oscillations. Again, the input Force of the Hand of the Operator has been taken as equal to 5 N.
The Simulink model used:



Then, computing the response of θ with few values of cT, we get:

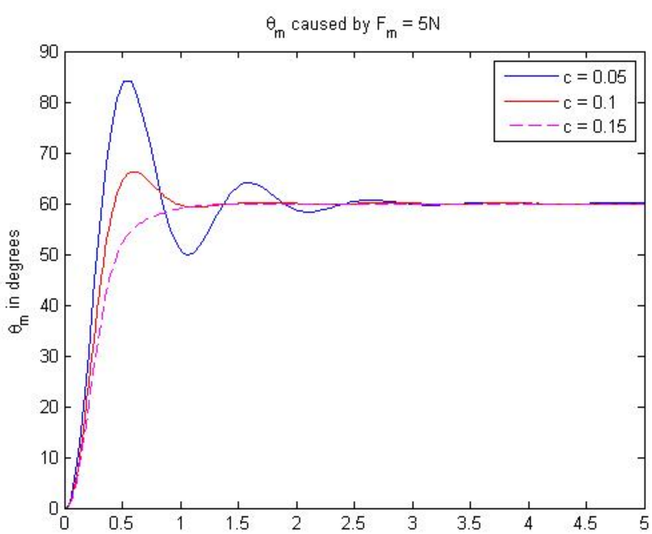


Figure 3: Finding cT value

By fine tuning, the required value has been found: cT=0.1 [N.sec/m].

2.2 Slave System Modelling

The Slave System is modelled exactly same as the Master System has been modelled in section 2.1, while the forces provided by the Operator for the Master are provided by the Environment for the Slave and the modelling of the hand of the Operator is kept for the Slave and refers to a transmission which would exist in the Slave System.

3 Analysis of the centralized system

We will know perform analysis of the centralized system: the Master and the Slave will be controlled by the same MIMO Controller. This analysis will be performed without any time delay.
Several criterions will be checked for this purpose:
- Criterion 1: Coupling between the master and slave direclty
- Criterion 2: Coupling between the master and slave via a Virtual Mass System.

3.1 Criterion 1: Coupling between the master and slave direclty

The Block Diagram of the System is:

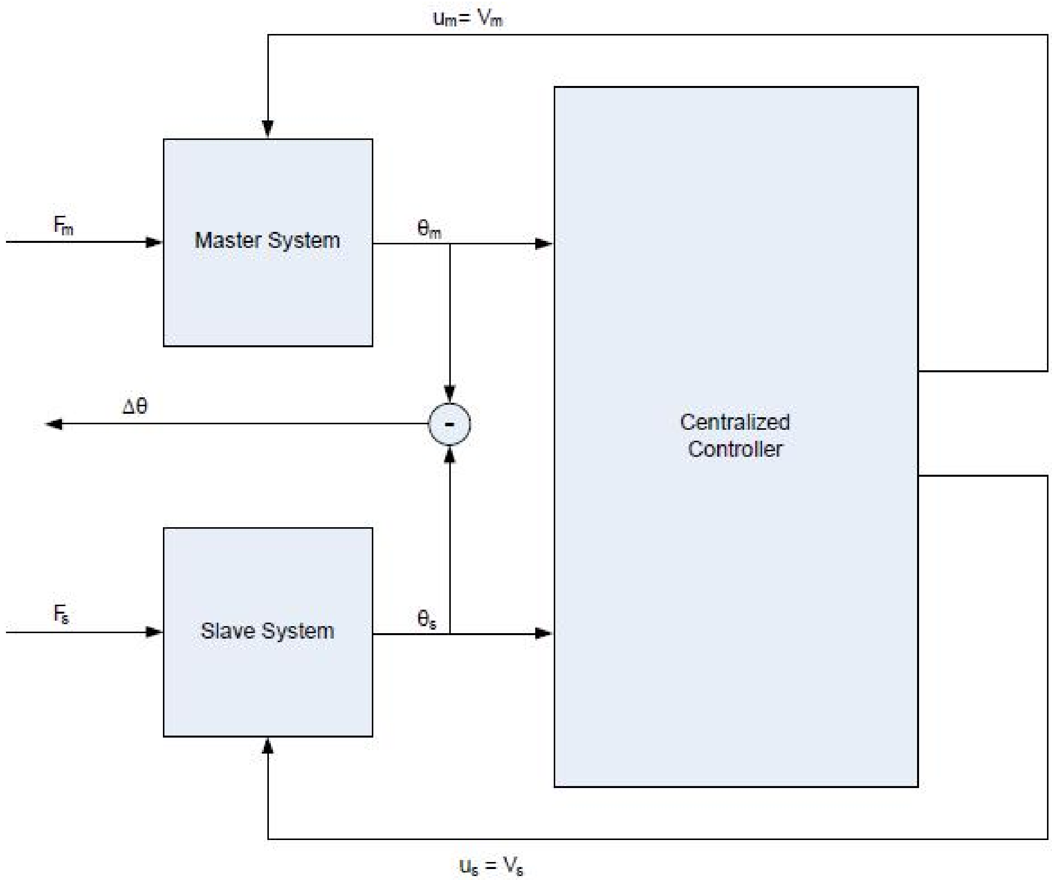


Figure 4: Criterion 1 - Block Diagram

So, the I/O of this system are defined as:

a. $z = [\Delta\theta, u_m, u_s]'$, while $\Delta\theta = \theta_m - \theta_s$

b. $w = [F_{0m}, F_{0s}]'$

c. $y = [\theta_m, \theta_s]'$

d. $u = [V_M, V_S]'$

And the Generalized Plant shall reflect:

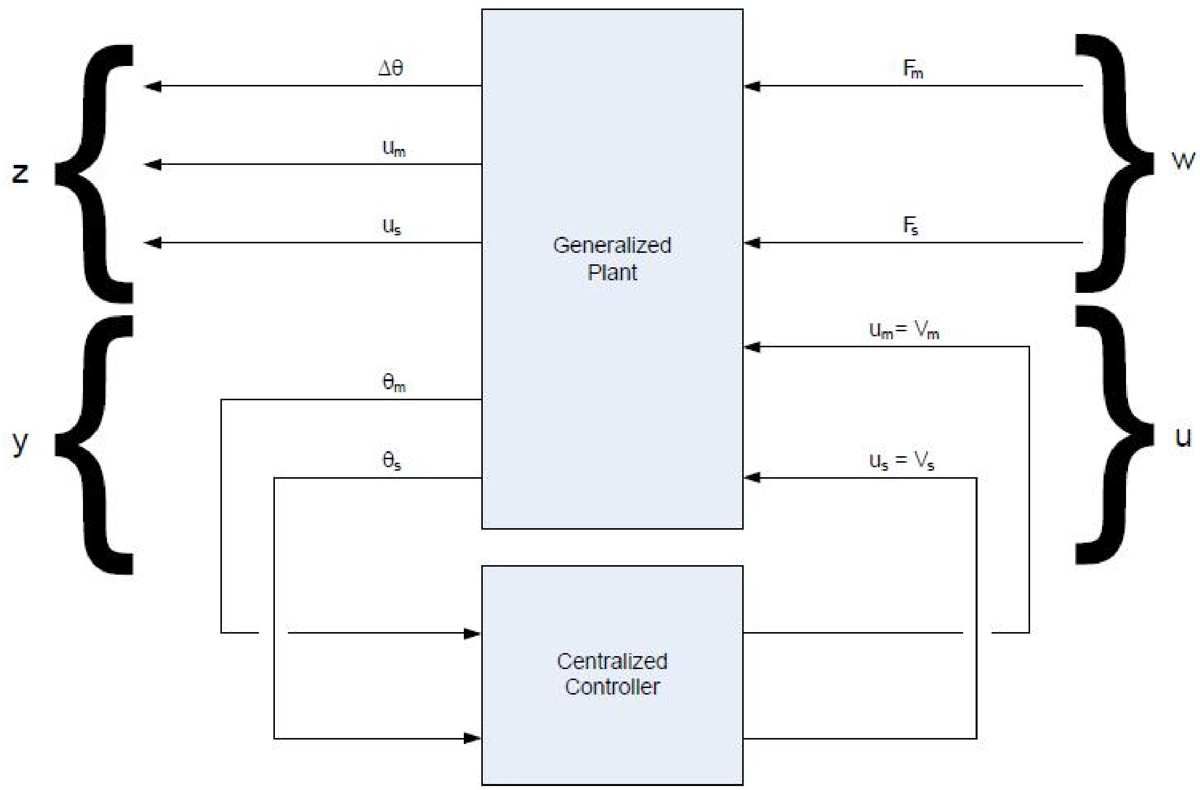


Figure 5: Criterion 1 - Generalized Plant

So, the resulting Generalized Plant is:

$$G_{GP}^{(1)} = \begin{bmatrix} G_{M_{thF}} & -G_{S_{thF}} & G_{M_{thV}} & -G_{S_{thV}} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ G_{M_{thF}} & 0 & G_{M_{thV}} & 0 \\ 0 & G_{S_{thF}} & 0 & G_{S_{thV}} \end{bmatrix}$$

In order to synthesize the Optimal Controller for this criterion, we will use the H infinity synthesis, by using the hinfsyn function in Matlab.

Let's recall that in order to be sure that the Controller synthesized by the hinfsyn function of Matlab is effective, the system must verify four assumptions (while A,B,C,D are the state space matrices of the system):
1. A1 - (A,B) stabilizable - this is checked by: rank(ctrb(A,B)) = 4 - OK
2. A2 - (A,C) detectable - this is checked by: rank(ctrb(A,C')) = 4 - OK
3. A3 - $D'_{12} * D12 = I$ - OK
4. A4 - $D'_{21} * D213.A3 \neq I$ - NOT OK!

In order to work with a conventional system, we must modify the system so that the fourth assumption will be checked.
Adding noise in the measurements of θ_m and θ_s shall solve this issue. Moreover, this is also more realistic to consider noises of the measurements.
This addition modifies the w and y vectors of the Generalized Plant:
$w = [F_{0m}, F_{0s}, n_{\theta_m}, n_{\theta_s}]'$
$y = [\theta_m + n_{\theta_m}, \theta_s + n_{\theta_s}]'$

The Generalized Plant with the a.m. noises becomes:

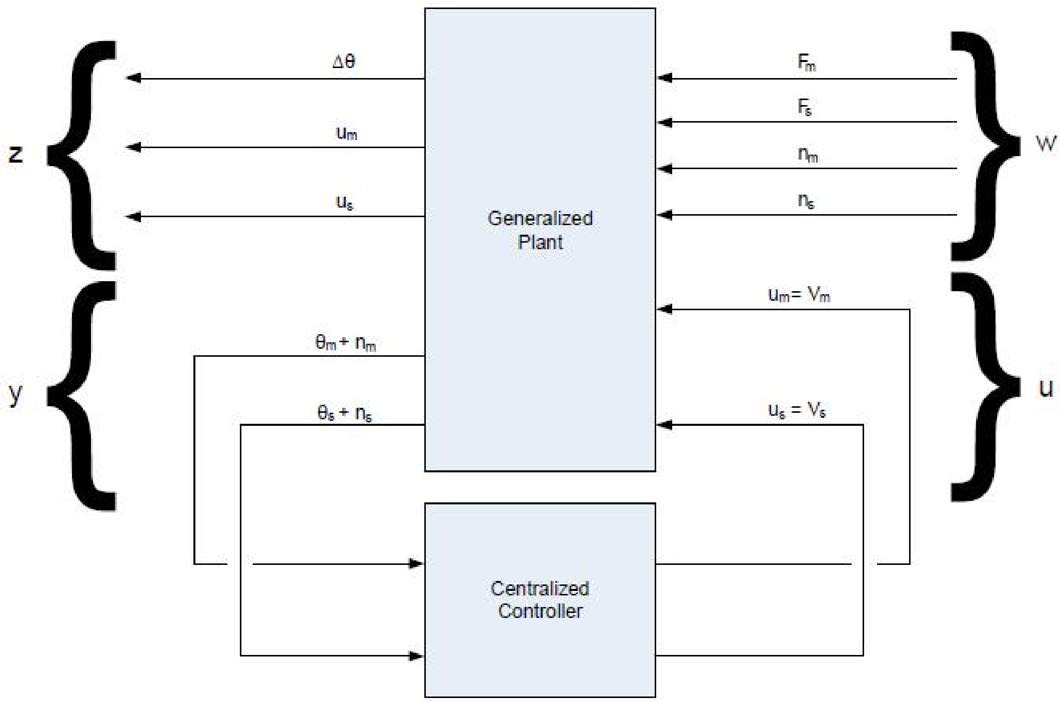


Figure 6: Criterion 1 - Generalized Plant with noises

The resulting Matrix for the Generalized Plant is:

$$G_{GP}^{(1)} = \begin{bmatrix} G_{M_{thF}} & -G_{S_{thF}} & 0 & 0 & G_{M_{thV}} & -G_{S_{thV}} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ G_{M_{thF}} & 0 & 1 & 0 & G_{M_{thV}} & 0 \\ 0 & G_{S_{thF}} & 0 & 1 & 0 & G_{S_{thV}} \end{bmatrix}$$

Let's try to use this Generalized Plant without any Weight functions on the minimized outputs (z).
Using the H infinity synthesis - hinfsyn function in matlab - we get the following Controller:

$$C^{(1)} = \frac{0.014611}{(s + 0.8588)} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

Plotting the step responses:

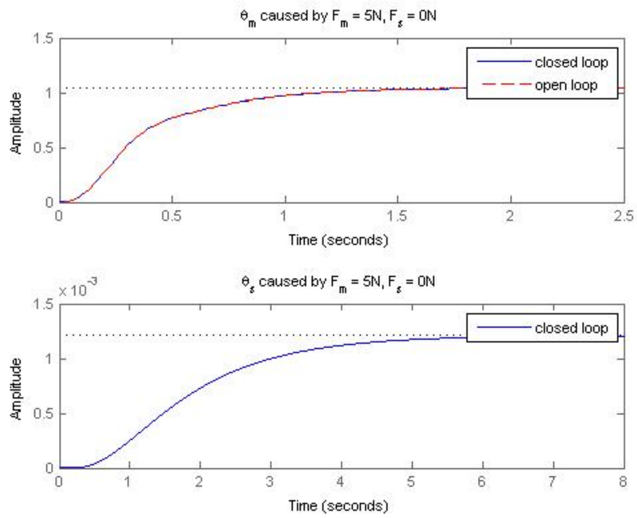


Figure 7: θ_m and θ_s caused by Force of the Operator's hand

The response of θ_m is the same in Open Loop and in Closed Loop. Therefore, the Controller "does not work".

We must add weight functions, so that the Controller will consider the $\Delta\theta$ more important to minimize than the Control Outputs u_m and u_s.
We will also add weight functions for the noises of the angles' measurements because noises are known to be significant only in high frequencies. The weight functions on $\Delta\theta$ will be chosen as a Low Pass Filter so that it would be significant only for low frequencies. Obviously, the Controller cannot be able to reach the required coupling in too high frequencies.

After adding weight functions:
$W_{\Delta\theta} = (s + 300)/(s + 1)$
$W_{u_m} = W_{u_s} = 0.2$
$W_{n_{\theta_m}} = W_{n_{\theta_s}} = (s + 1)/(10 * (s + 1000))$

The bode amplitude diagram of the weight on the coupling target $\Delta\theta$ is then:

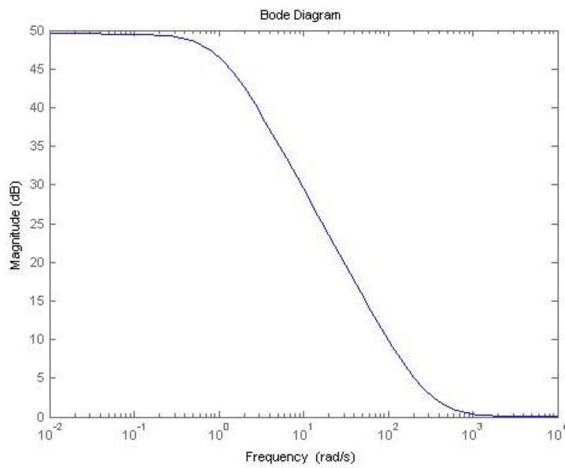


Figure 8: $W_{\Delta\theta}$ Bode amplitude diagram

And the bode amplitude diagram of the weight on the noise is then:

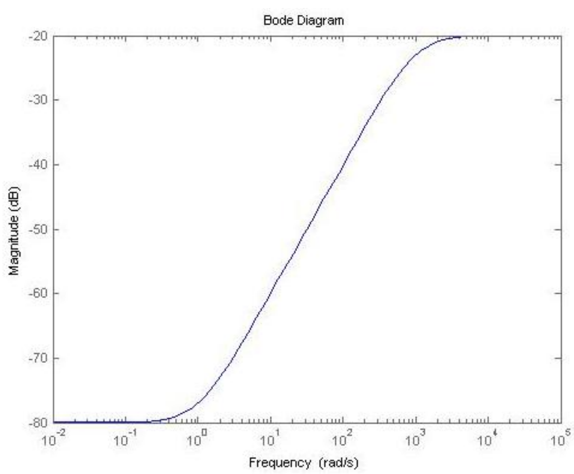


Figure 9: $W_{n_{\theta_m}}$ and $W_{n_{\theta_s}}$ Bode amplitude diagram

The Generalized Plant becomes:

$$G_{GP}^{(1)} = \begin{bmatrix} W_{\Delta\theta}G_{M_{thF}} & -W_{\Delta\theta}G_{S_{thF}} & 0 & 0 & W_{\Delta\theta}G_{M_{thV}} & -W_{\Delta\theta}G_{S_{thV}} \\ 0 & 0 & 0 & 0 & W_{u_m} & 0 \\ 0 & 0 & 0 & 0 & 0 & W_{u_s} \\ G_{M_{thF}} & 0 & W_{n_{\theta_m}} & 0 & G_{M_{thV}} & 0 \\ 0 & G_{S_{thF}} & 0 & W_{n_{\theta_s}} & 0 & G_{S_{thV}} \end{bmatrix}$$

Using the H infinity synthesis - hinfsyn function in matlab - we get the following Controller:

$$C^{(1)} = \frac{2505.658(s+1000)(s+346)(s+7.178)(s^2+12.08s+147.9)}{(s+352)(s+1)(s^2-22.9s+298.4)(s^2+180.2s+1.57e04)} \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$$

3.1.1 With $F_m = 5N$ and $F_s = 0N$

Plotting the step responses:

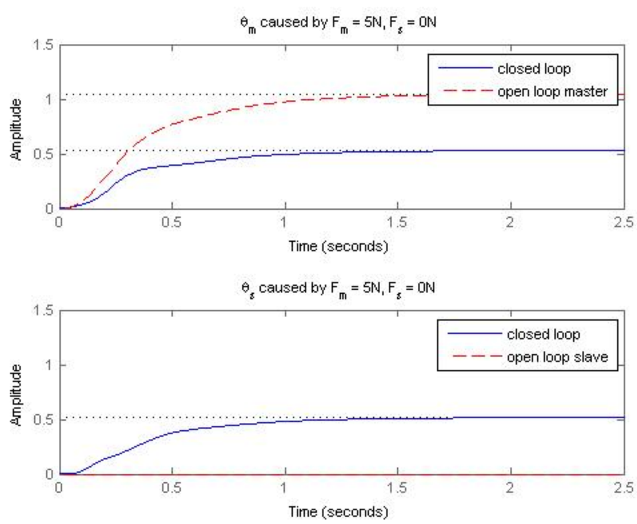


Figure 10: Criterion 1 - $F_m = 5N$ and $F_s = 0N$ - θ_m and θ_s

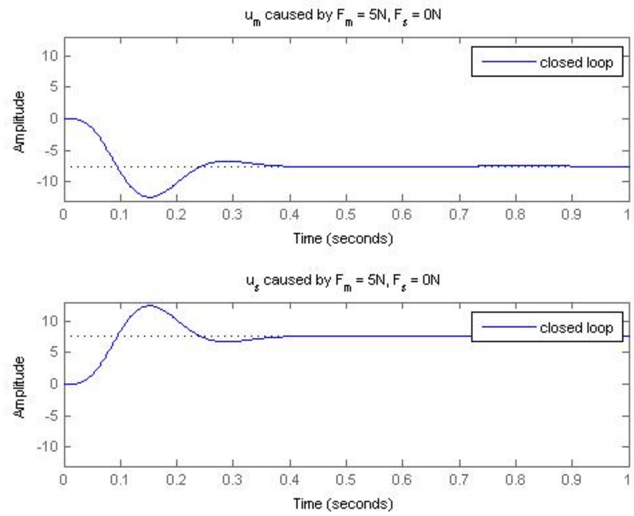


Figure 11: Criterion 1 - $F_m = 5N$ and $F_s = 0N$ - u_m and u_s

In order to check that the criterion for minimizing $\Delta\theta$ has been solved, let's plot θ_m and θ_s on the same graph:

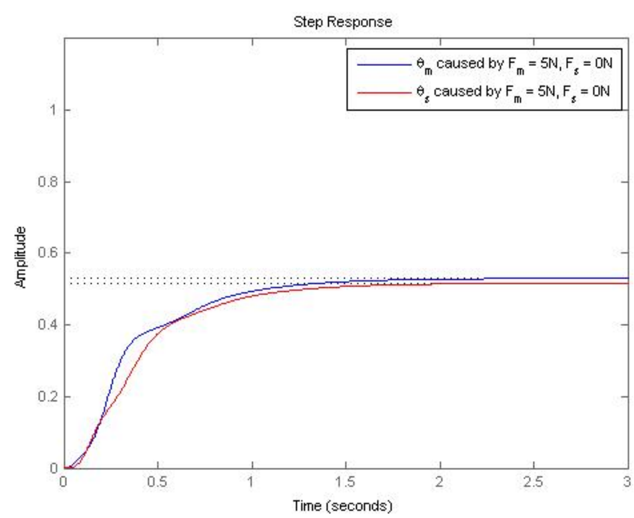


Figure 12: Criterion 1 - $F_m = 5N$ and $F_s = 0N$ - Both θ_m and θ_s

When looking at the plots of θ_m and θ_s , we can see that the closed loop system behaves differently than the open loop system. Since our criterion requires the closed loop behaviour of θ_m and the closed loop behaviour of θ_s to be equal. Then, the Controller "creates" a new system which the Master and the Slave "sense" to be the real system: this system will be named as the effective system.
Let's model this system as we modelled the master (and the slave): a Joystick with its DC motor and a force applied at the opposite end of the joystick, by the same modelled hand (simple mass-spring-damper system). The parameters of the "hand's system" will be kept. Let's find the parameters which characterize the joystick itself, in terms of J_e, c_e, k_e.

The Force applied on this effective system would be the vectorial sum of the Master's and the Slave's forces.
Therefore, in the previous simulation: F_e=5N.

Intuitive guess: k_e=k_m+k_s
Taking:
- $J_e = 0.5 * (J_m + J_s)$ (found by trial and error)
- $c_e = c_m$+c_s

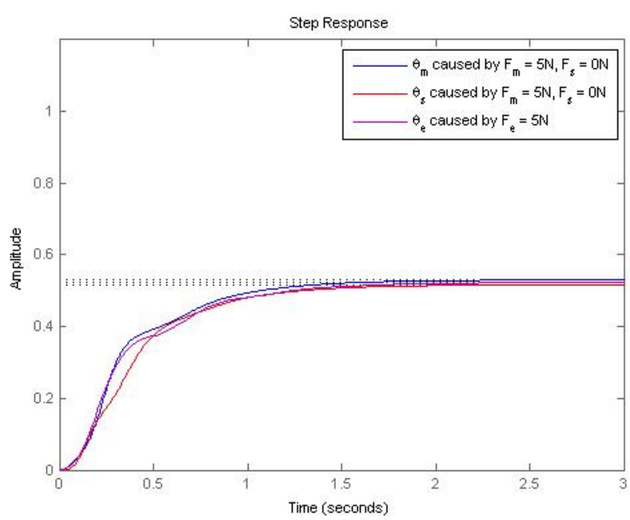


Figure 13: Criterion 1 - $F_m = 5N$ and $F_s = 0N$ - θ_e

The greatest Control Output that can be used with our experimental system is $V_{max} = 12V$.
This is the maximum Control Output that we got in the previous simulation, so we will not be able to get better behavior of the system with this criterion.
Let's check the behavior of the system with several inputs, i.e. varying the values of F_m and F_s:

3.1.2 With $F_m = F_s = 5N$

Plotting the step responses:

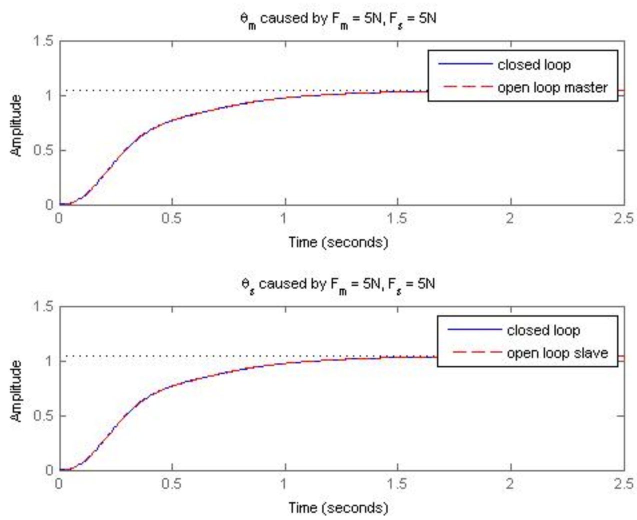


Figure 14: Criterion 1 - $F_m = 5N$ and $F_s = 5N$ - θ_m and θ_s

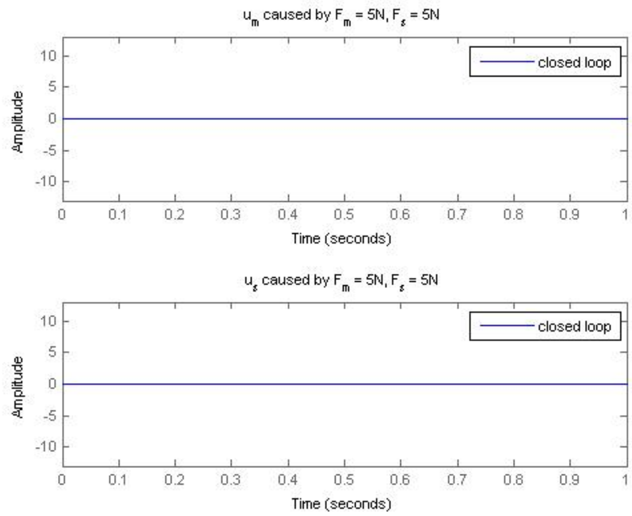


Figure 15: Criterion 1 - $F_m = 5N$ and $F_s = 5N$ - u_m and u_s

In order to check that the criterion for minimizing $\Delta\theta$ has been solved, let's plot θ_m and θ_s on the same graph, together with the effective System in Open Loop:

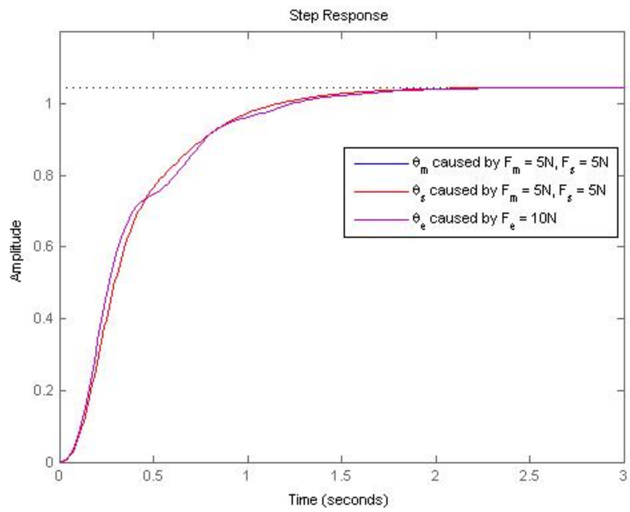


Figure 16: Criterion 1 - $F_m = 5N$ and $F_s = 5N$ - θ_e

In the above case, the forces applied on the master's joystick (by the operator's hand) and on the slave's joystick (by the environnement) are equal. This is the reason why the Control Output are so "tiny": because of the symetry of our case, even in Open Loop, the responses are equal: the Controller does not have anything to do at all (also explains why the open loop and closed loop responses are fitting and why the θ_m and θ_s are exactly the same).

3.1.3 With $F_m = 2$ and $F_s = 5N$

Plotting the step responses:

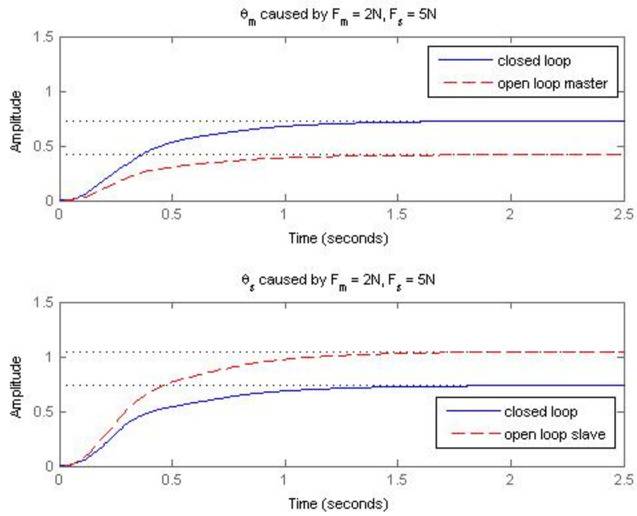


Figure 17: Criterion 1 - $F_m = 2N$ and $F_s = 5N$ - θ_m and θ_s

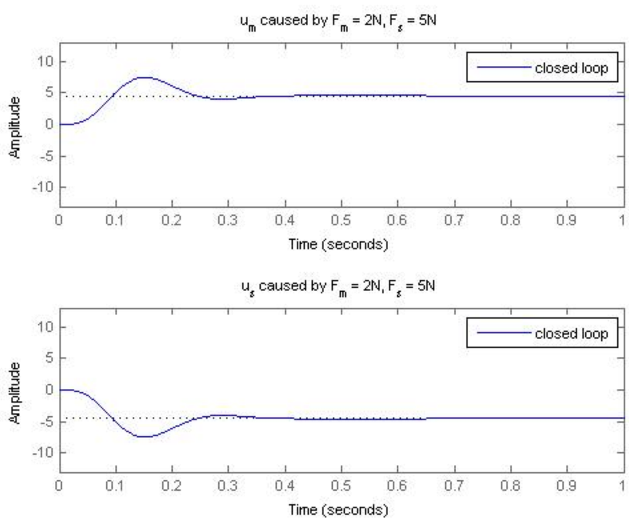


Figure 18: Criterion 1 - $F_m = 2N$ and $F_s = 5N$ - u_m and u_s

In order to check that the criterion for minimizing $\Delta\theta$ has been solved, let's plot θ_m and θ_s on the same graph, together with the Effective System in Open Loop :

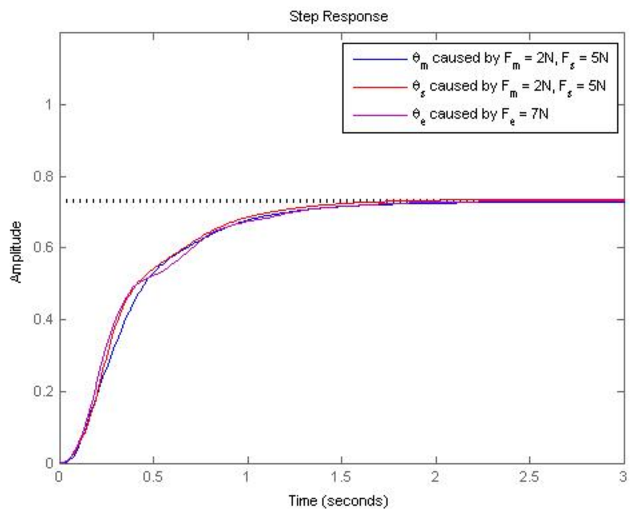


Figure 19: Criterion 1 - $F_m = 2N$ and $F_s = 5N$ - θ_e

In the above simulation, the Controller has reached an almost perfect coupling between the master and the slave systems, while the Control Outputs are kept below the limit of 12V.

3.1.4 With $F_m = F_s = 5N$, with time delay

This time, the force applied on the slave system - F_s - will be applied with a time delay of $t = 1.5sec$, which seems to be close to settling time:

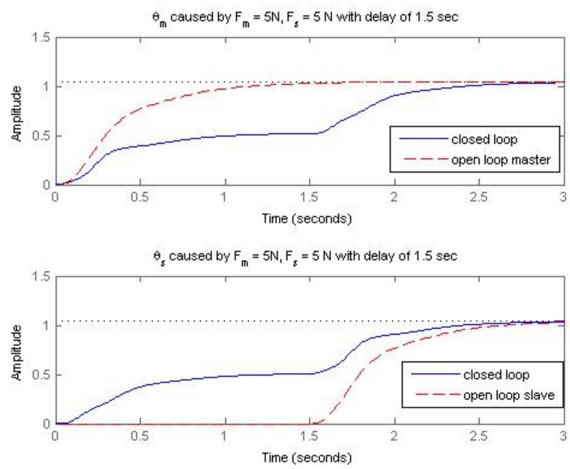


Figure 20: Criterion 1 - $F_m = 5N$ and $F_s = 5N$ delayed by 1.5 sec - θ_m and θ_s

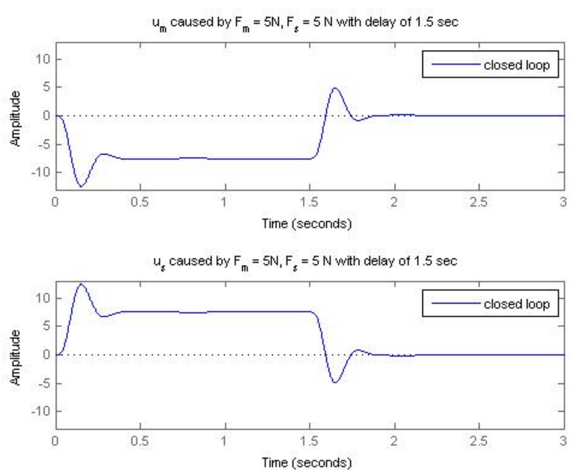


Figure 21: Criterion 1 - $F_m = 5N$ and $F_s = 5N$ delayed by 1.5 sec - u_m and u_s

In order to check that the criterion for minimizing $\Delta\theta$ has been solved, let's plot θ_m and θ_s on the same graph, together with the Effective System in Open Loop:

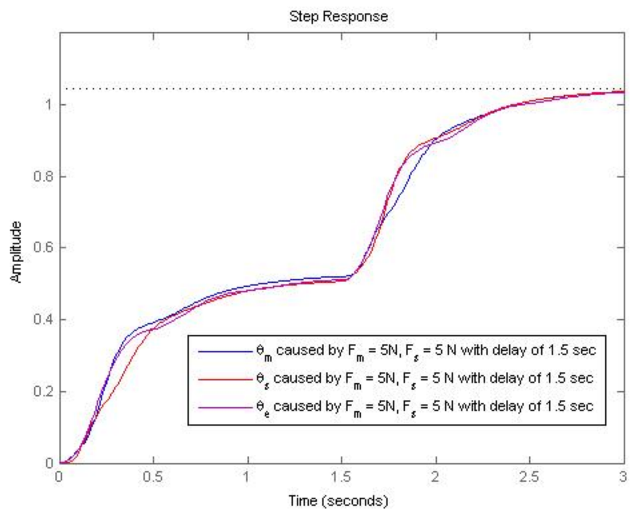


Figure 22: Criterion 1 - $F_m = 5N$ and $F_s = 5N$ delayed by 1.5 sec - θ_e

Again, in the above simulation, the Controller has reached an almost perfect coupling between the master and the slave systems, while the Control Outputs are kept below the limit of 12V.

3.1.5 With $F_m = 3N$ and $F_s = -1N$

Plotting the step responses:

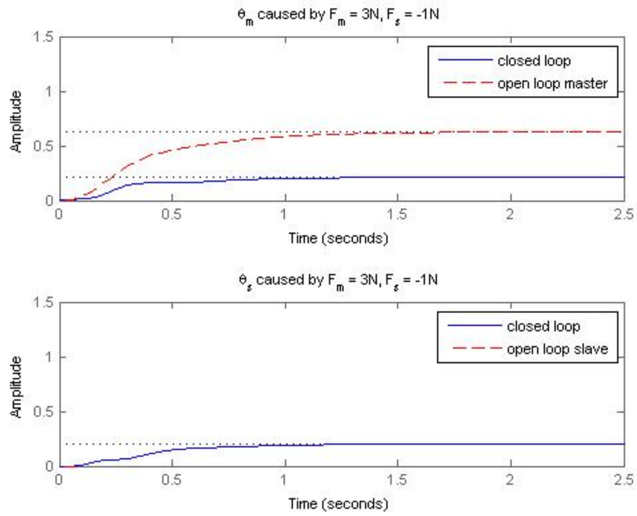


Figure 23: Criterion 1 - $F_m = 3N$ and $F_s = -1N$ - θ_m and θ_s

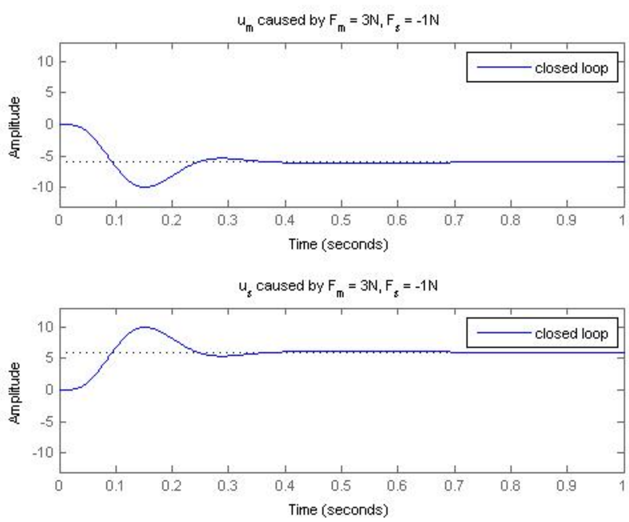


Figure 24: Criterion 1 - $F_m = 3N$ and $F_s = -1N$ - u_m and u_s

In order to check that the criterion for minimizing $\Delta\theta$ has been solved, let's plot θ_m and θ_s on the same graph, together with the Effective System in Open Loop:

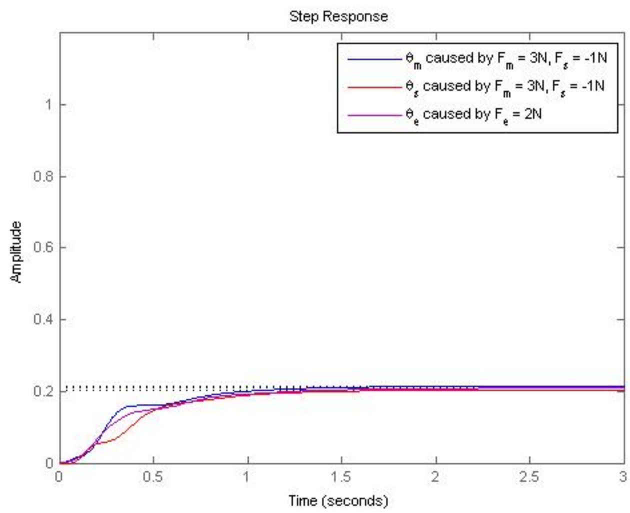


Figure 25: Criterion 1 - $F_m = 3N$ and $F_s = -1N$ - θ_e

In this case also, the Controller has reached an almost perfect coupling between the master and the slave systems, while the Control Outputs are kept below the limit of 12V. Nevetheless, in order to keep this Control Outputs' limitation, the forces' amplitude had to be reduced (from 5N in previous simulations to 3N and 1N).

3.1.6 With $F_m = 2N$ and $F_s = -2N$

Plotting the step responses:

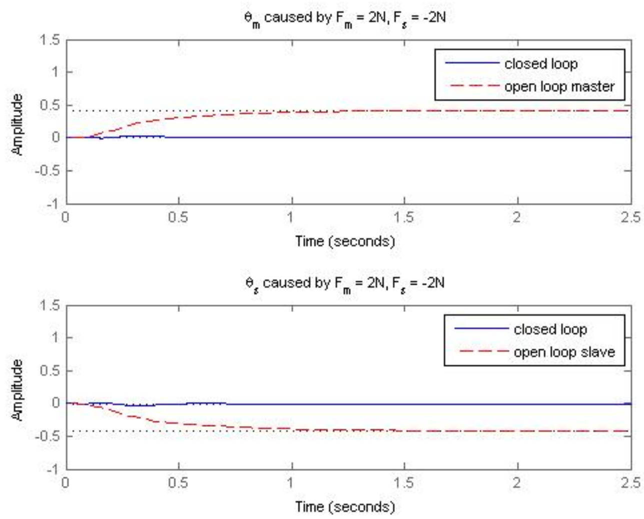


Figure 26: Criterion 1 - $F_m = 2N$ and $F_s = -2N$ - θ_m and θ_s

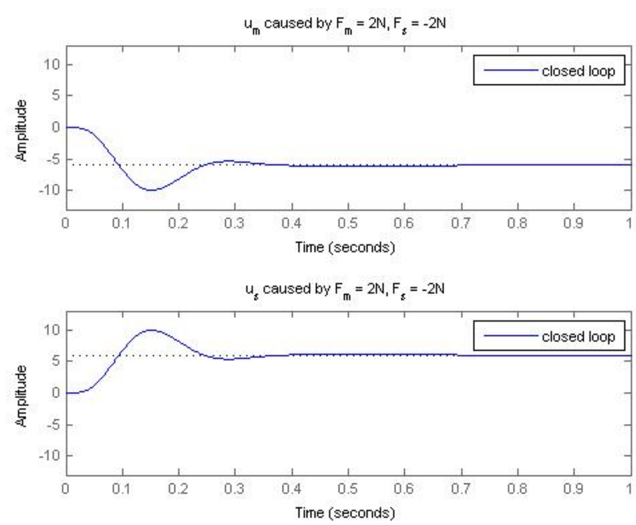


Figure 27: Criterion 1 - $F_m = 2N$ and $F_s = -2N$ - u_m and u_s

In order to check that the criterion for minimizing $\Delta\theta$ has been solved, let's plot θ_m and θ_s on the same graph, together with the Effective System in Open Loop:

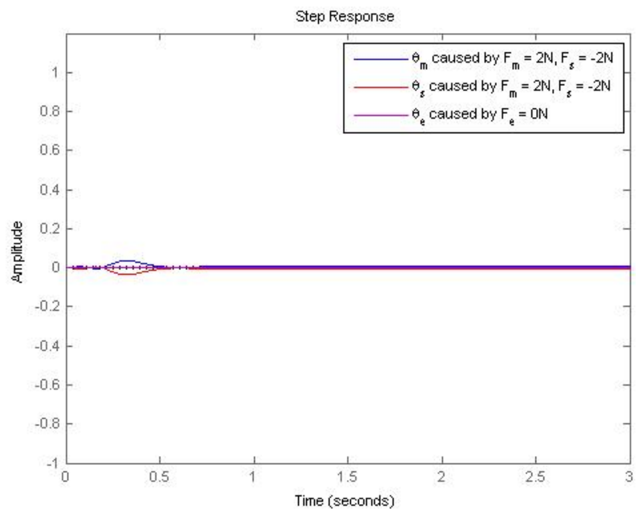


Figure 28: Criterion 1 - $F_m = 2N$ and $F_s = -2N$ - θ_e

In the above graph, the plot of the Open Loop of the resulting effective System is not relevant because the force F_e applied on the effective System is equal to zero.
The Controller has reached a perfect coupling between the master and the slave systems, while the Control Outputs are kept below the limit of 12V.
The forces applied on the Master and Slave systems have been taken again with reduced amplitude (from 5N in previous amplitude to 2N in this simulation), in order to keep the Control Outputs' limitation.

3.1.7 With $F_m = 2N$ and $F_s = -2N$, with time delay

With $F_m = 2N$ and $F_s = -2N$, but this time, F_s will be applied with a time delay of $t = 1.5sec$, which seems to be close to settling time.

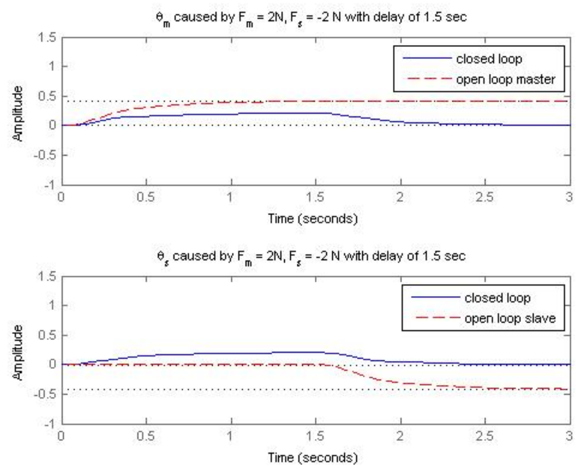


Figure 29: Criterion 1 - $F_m = 2N$ and $F_s = -2N$ delayed by 1.5 sec - θ_m and θ_s

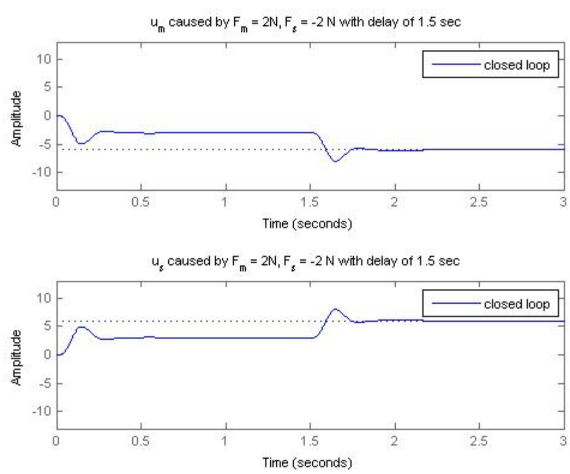


Figure 30: Criterion 1 - $F_m = 2N$ and $F_s = -2N$ delayed by 1.5 sec - u_m and u_s

In order to check that the criterion for minimizing $\Delta\theta$ has been solved, let's plot θ_m and θ_s on the same graph,together with the effective System in Open Loop:

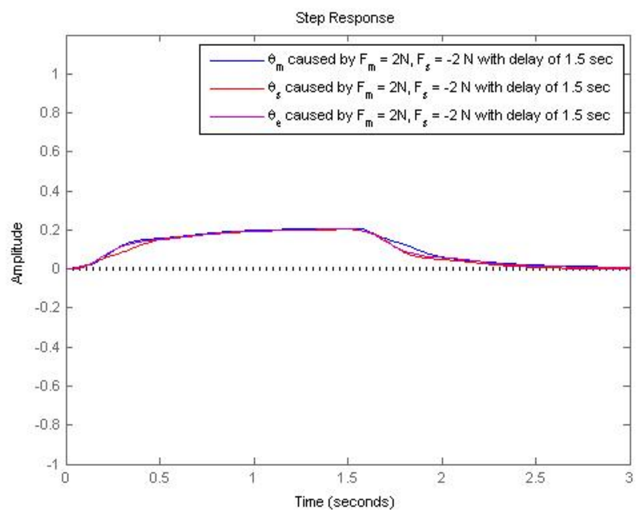


Figure 31: Criterion 1 - $F_m = 2N$ and $F_s = -2N$ delayed by 1.5 sec - θ_e

In the above simulation, the Controller has reached an almost perfect coupling between the master and the slave systems, while the Control Outputs are kept below the limit of 12V.
The forces applied on the Master and Slave systems have been taken again with reduced amplitude (from 5N in previous amplitude to 2N in this simulation), in order to keep the Control Outputs' limitation.

3.1.8 With $F_m = 5N$ and $F_s = 0N$, with less aggressive controller

Let's see the behavior of the system with a less aggressive controller, i.e. when maximal Control Output is 9 V instead of 12 V:
For this purpose, the weight factors of the Control Output have been modified so that: $W_{u_m} = W_{u_s} = 2$.

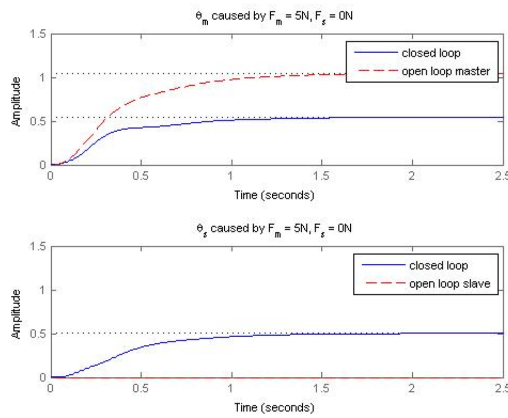


Figure 32: Criterion 1 - $F_m = 5N$ and $F_s = 0N$ with less aggressive controller - θ_m and θ_s

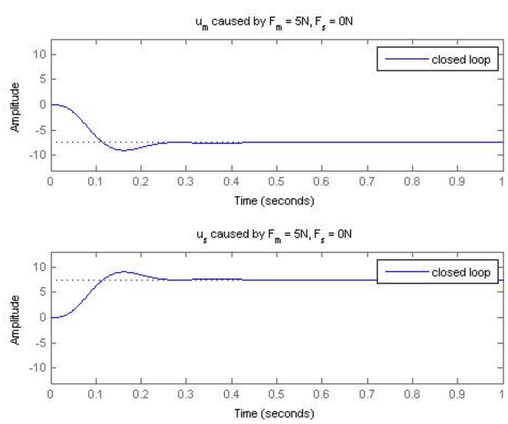


Figure 33: Criterion 1 - $F_m = 5N$ and $F_s = 0N$ with less aggressive controller - u_m and u_s

In order to check that the criterion for minimizing $\Delta\theta$ has been solved, let's plot θ_m and θ_s on the same graph, together with the effective System in Open Loop:

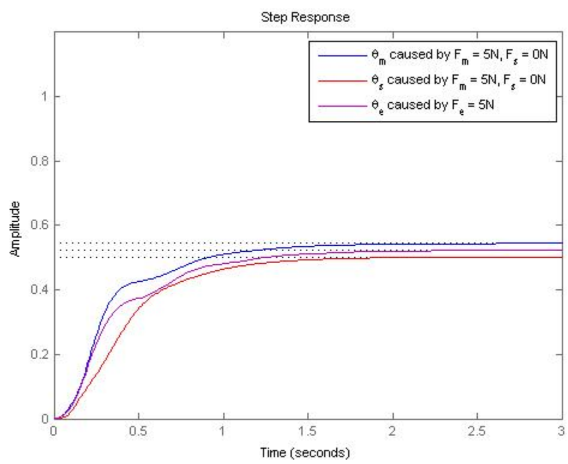
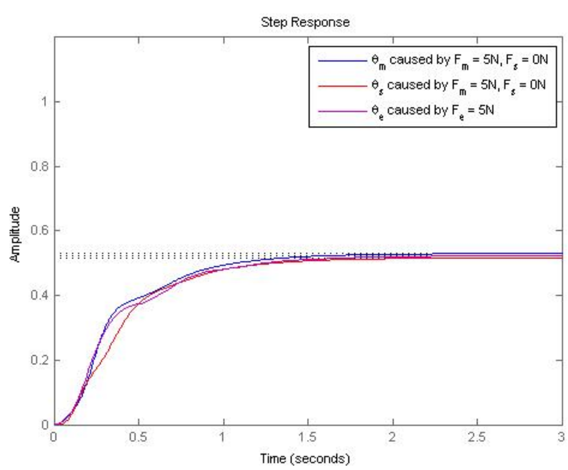


Figure 34: Criterion 1 - $F_m = 5N$ and $F_s = 0N$ with less aggressive controller - θ_e

Recalling the same graph with the maximum Control Output of 12 V:



3.1.9 Criterion 1 - Conclusions

According to the above simulations, the Controller does reach the criteria and $\Delta\theta$ is closed to zero: almost perfect coupling is achieved in all cases - except for the case with the less aggressive Controller.
Nevertheless, looking at the graphs of θ_m and θ_s in open and closed loop, one can see that the controller makes the system very similar to the system that has been defined above and called the Effective System.
This Effective System is constant: its parameters are constant and cannot be modified with this Control optimization criterion.
Therefore, one can work with this criterion is order to reach telepresence, but the stiffness and damping of the closed loop system cannot be modified and this restriction might be a handicap is some cases.

3.2 Criterion 2: Coupling between the master and slave via a Virtual Mass System

This means that instead of minimizing $\Delta\theta = \theta_m - \theta_s$, we will define a virtual system and we will try to minimize the differences between:
- the responses of the master system and the virtual mass system,
- the responses of the slave system and the virtual mass system.

The Block Diagram of the System is:

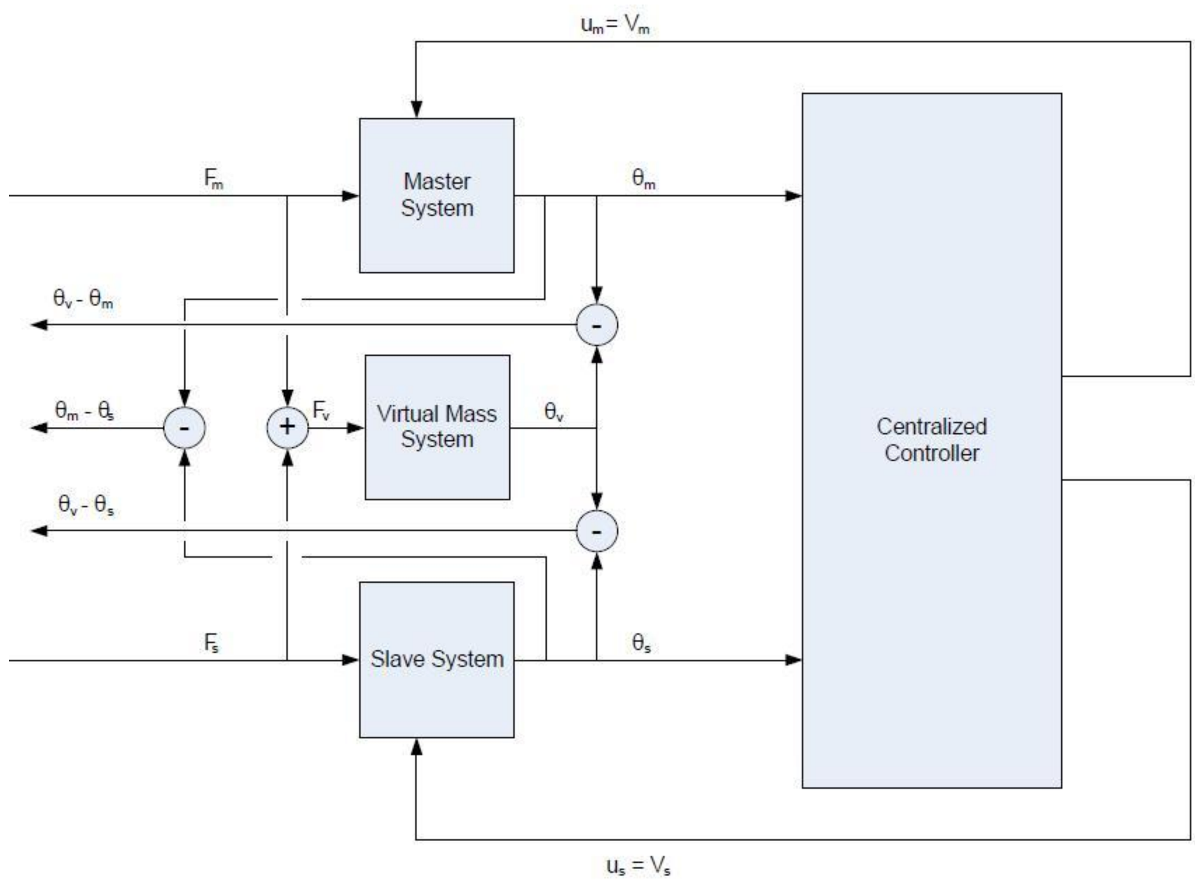


Figure 35: Criterion 2 - Block Diagram

So, the I/O of this system are defined as:
a. $z = [\Delta\theta_{vm}, \Delta\theta_{vs}, \Delta\theta_{ms}, u_m, u_s]'$,
while $\Delta\theta_{vm} = \theta_v - \theta_m$, $\Delta\theta_{vs} = \theta_v - \theta_s$,$\Delta\theta_{ms} = \theta_m - \theta_s$.
b. $w = [F_{0m}, F_{0s}]'$
c. $y = [\theta_m, \theta_s]'$
d. $u = [V_m, V_s]'$

And the Generalized Plant shall reflect:

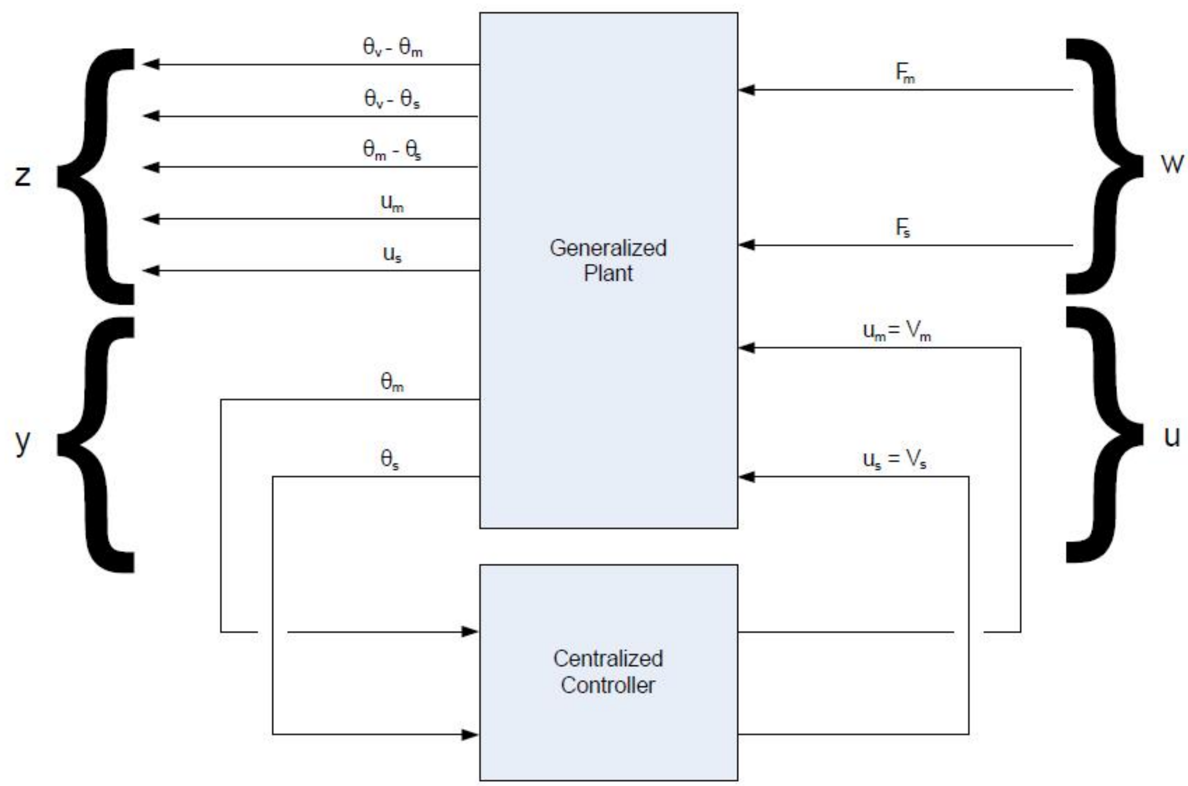


Figure 36: Criterion 2 - Generalized Plant

Then, the Generalized Plant is:

$$G_{GP}^{(2)} = \begin{bmatrix} G_{\theta_v,F_m} - G_{\theta_m,F_m} & G_{\theta_v,F_s} & -G_{\theta_m,V_m} & 0 \\ G_{\theta_v,F_m} & G_{\theta_v,F_s} - G_{\theta_s,F_s} & 0 & -G_{\theta_s,V_s} \\ G_{\theta_m,F_m} & -G_{\theta_s,F_s} & G_{\theta_m,V_m} & -G_{\theta_s,V_s} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ G_{\theta_m,F_m} & 0 & G_{\theta_m,V_m} & 0 \\ 0 & G_{\theta_s,F_s} & 0 & G_{\theta_s,V_s} \end{bmatrix}$$

First of all, one shall remark that the degree of the Generalized Plant in this criterion is greater than in the first criterion.
We shall know define the Virtual Mass system:
Let's consider it as a joystick - similar to the master's joystick.
The advantage with this criterion is that we can decide what will be the system's parameters, i.e. the stiffness and damping of the system can be chosen by the designer.
The design of the Controller will set the common system - the Virtual Mass system - that the Master and the Slave will "try to look as".
Joystick parameters (same as the master's joystick):
$L_v = L_m$ [m]
$kT_v = kT_m (= kT_s)$ [N/rad]
$cT_v = cT_m + cT_s$ [N.sec/rad]
$J_v = 0.5 * (J_m + J_s)$ [kg.m^2]
These will be changed in the following simulations.

Then, the transfer functions are the same as the transfer functions written for the master's joystick.
In order to synthesize the Optimal Controller for this criterion, we will use the H infinity synthesis, by using the hinfsyn function in Matlab.

Let's recall that in order to be sure that the Controller synthesized by the hinfsyn function of Matlab is effective, the system must verify four assumptions (while A,B,C,D are the state space matrices of the system):
1. A1 - (A,B) stabilizable - this is checked by: rank(ctrb(A,B)) = 4 - OK
2. A2 - (A,C) detectable - this is checked by: rank(ctrb(A,C')) = 4 - OK
3. A3 - $D'_{12} * D12 = I$ - OK
4. A4 - $D'_{21} * D213.A3 \neq I$ - NOT OK!
As done for the first Criterion, let's add the measurements' noises for θ_m and θ_s.

This addition modifies the w and y vectors of the Generalized Plant:
$w = [F_{0m}, F_{0s}, n_{\theta_m}, n_{\theta_s}]'$
$y = [\theta_m + n_{\theta_m}, \theta_s + n_{\theta_s}]'$

The Generalized Plant with the a.m. noises becomes:

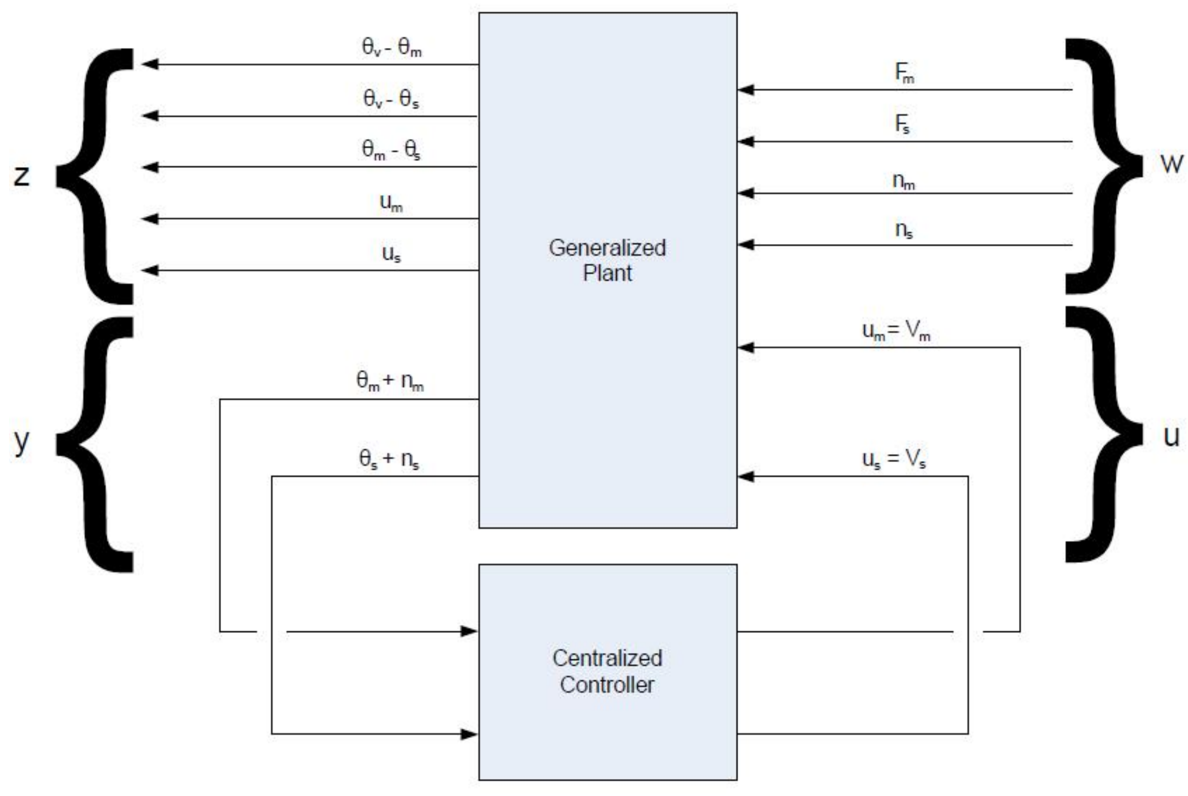


Figure 37: Criterion 2 - Generalized Plant with noises

The Generalized Plant becomes:

$$G_{GP}^{(2)} = \begin{bmatrix} G_{\theta_v,F_m} - G_{\theta_m,F_m} & G_{\theta_v,F_s} & 0 & 0 & -G_{\theta_m,V_m} & 0 \\ G_{\theta_v,F_m} & G_{\theta_v,F_s} - G_{\theta_s,F_s} & 0 & 0 & 0 & -G_{\theta_s,V_s} \\ G_{\theta_m,F_m} & -G_{\theta_s,F_s} & 0 & 0 & G_{\theta_m,V_m} & -G_{\theta_s,V_s} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ G_{\theta_m,F_m} & 0 & 1 & 0 & G_{\theta_m,V_m} & 0 \\ 0 & G_{\theta_s,F_s} & 0 & 1 & 0 & G_{\theta_s,V_s} \end{bmatrix}$$

We shall also add weight functions as done for the first criterion:
$W_{\Delta\theta_{vm}} = W_{\Delta\theta_{vm}} = W_{\Delta\theta_{ms}} = (s+300)/(s+1)$
$W_{u_m} = W_{u_s} = 12$
$W_{n_{\theta_m}} = W_{n_{\theta_s}} = (s+1)/(10*(s+1000))$

Then, perform simulations on the system:

With $F_m = 5N$ and $F_s = 0N$:
Plotting the step responses of the master and the slave, and checking their both coupling with the virtual system:

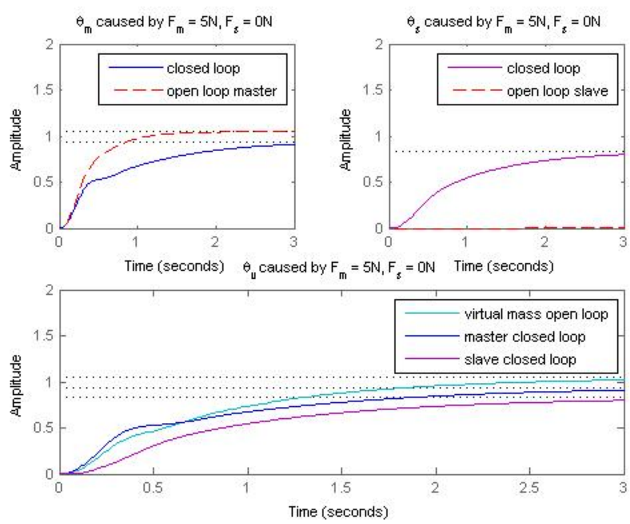


Figure 38: Criterion 2 - $F_m = 5N$ and $F_s = 0N$ - θ_v, θ_m and θ_s

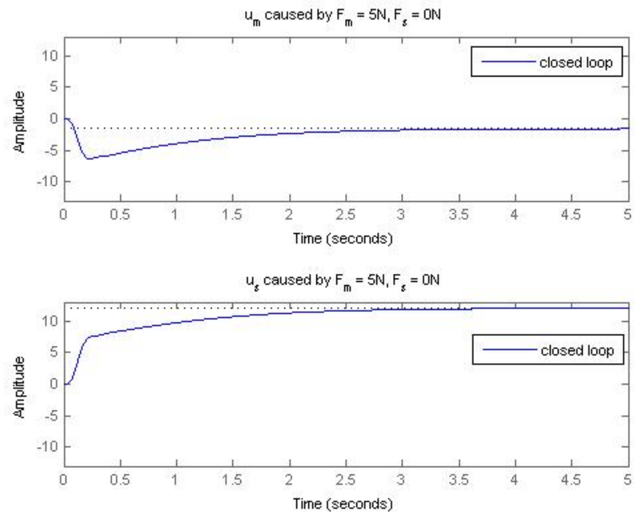


Figure 39: Criterion 2 - $F_m = 5N$ and $F_s = 0N$ - u_m and u_s

We can see that the controller manages to minimize the vector z of the system: θ_v, θ_m and θ_s are very close one to the other and almost equal in steady state.

Let's check the following cases:
1) Setting the stiffness of the Virtual Mass System kT_{VM} as the joystick's system stiffness kT_m, but with Moment of Inertia greater than the Joystick's: $J_{VM} > J_m$.
2) Setting stiffness of the Virtual Mass system smaller than the joystick's system: $kT_{VM} < kT_m$.
3) Setting the parameters of the Virtual Mass System equal to the effective system's parameters (the system found with criterion 1 analysis).

We will run simulations for:
- $F_m = 5N$ and $F_s = 0N$,
and
- $F_m = 5N$ and $F_s = -2N$.

3.2.1 First case: $J_{VM} > J_m$

Setting the stiffness as the Joystick system's stiffness kT_m, but with Moment of Inertia greater than the Joystick's: $J_{VM} > J_m$
Let's set: $J_{VM} = 10 * J_m$
Using the weight functions for both simulations:
$W_{\Delta\theta_{vm}} = W_{\Delta\theta_{vm}} = W_{\Delta\theta_{ms}} = (s + 300)/(s + 1)$
$W_{u_m} = W_{u_s} = 14$
$W_{n_{\theta_m}} = W_{n_{\theta_s}} = (s + 1)/(10 * (s + 1000))$

The Bode Diagrams (amplitude only) of the above weight functions are depicted in Figure 8 and in Figure 9.

With $F_m = 5N$ and $F_s = 0N$:

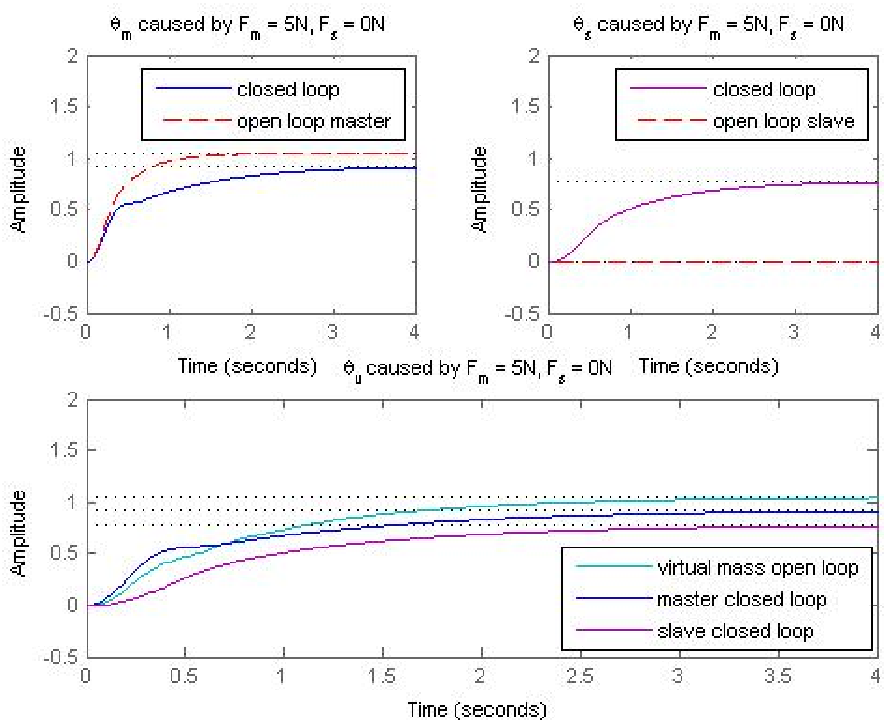


Figure 40: Criterion 2 - First case: $J_{VM} > J_m$ - $F_m = 5N$ and $F_s = 0N$ - θ_v, θ_m and θ_s

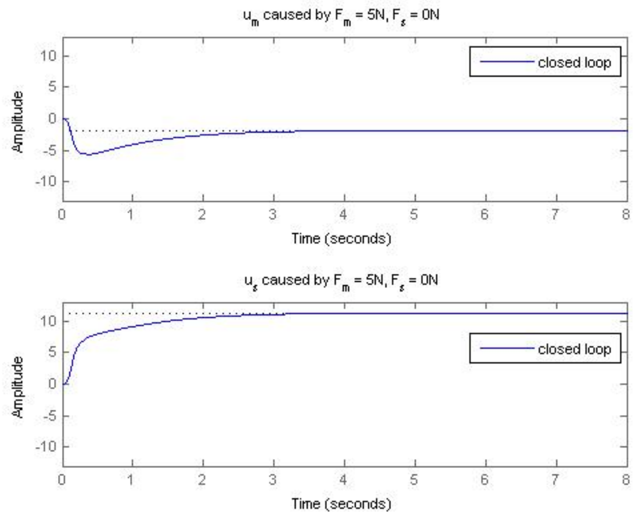


Figure 41: Criterion 2 - First case: $J_{VM} > J_m$ - $F_m = 5N$ and $F_s = 0N$ - u_m and u_s

With $F_m = 5N$ and $F_s = -2N$:

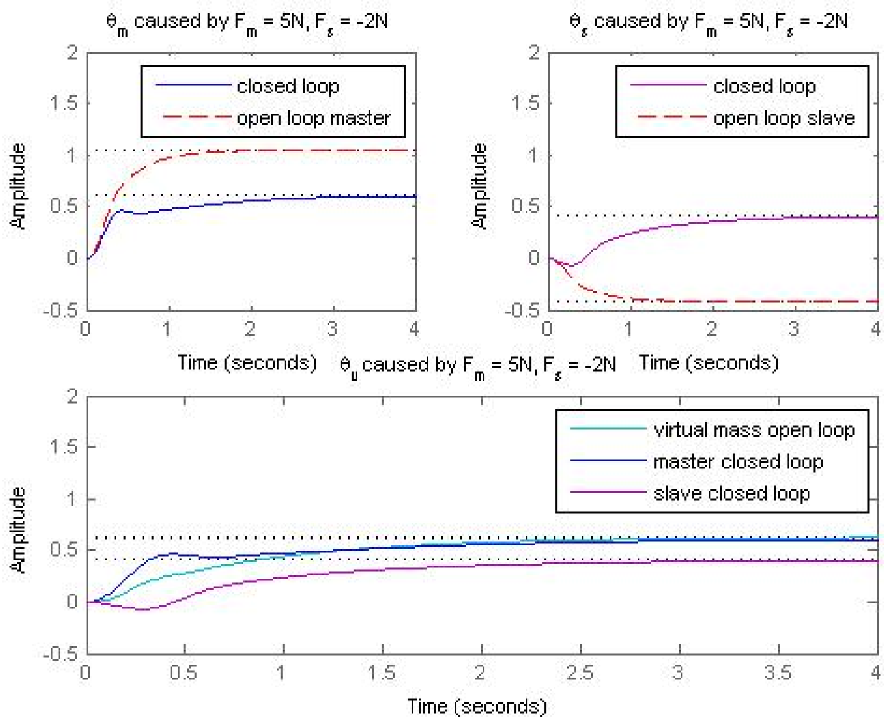


Figure 42: Criterion 2 - First case: $J_{VM} > J_m$ - $F_m = 5N$ and $F_s = -2N$ - θ_v, θ_m and θ_s

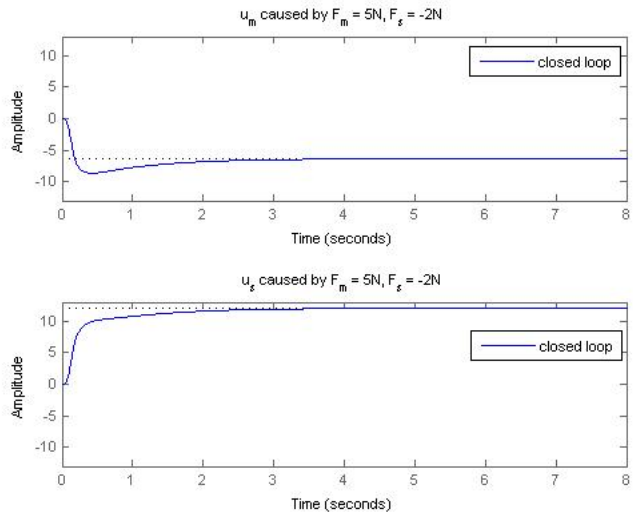


Figure 43: Criterion 2 - First case: $J_{VM} > J_m$ - $F_m = 5N$ and $F_s = -2N$ - u_m and u_s

In the above simulations, the Controller has reached a reasonable coupling (shall be confirmed by the specific system's designer) between the master and the slave systems, while the Control Outputs are kept below the limit of 12V.

3.2.2 Second case: $kT1_{VM} < kT_m$

We got: $kT_m = 0.48[N/rad]$, so let's set:
$kT1_{VM} = 0.48/2 = 0.24[N/rad]$.

The weight functions have been modified in order to match the design limitations for both simulations:
$W_{\Delta\theta_{vm}} = W_{\Delta\theta_{vm}} = W_{\Delta\theta_{ms}} = (s + 300)/(s + 1)$
$W_{u_m} = W_{u_s} = 26$
$W_{n_{\theta_m}} = W_{n_{\theta_s}} = (s + 1)/(10 * (s + 1000))$

With $F_m = 5N$ and $F_s = 0N$:

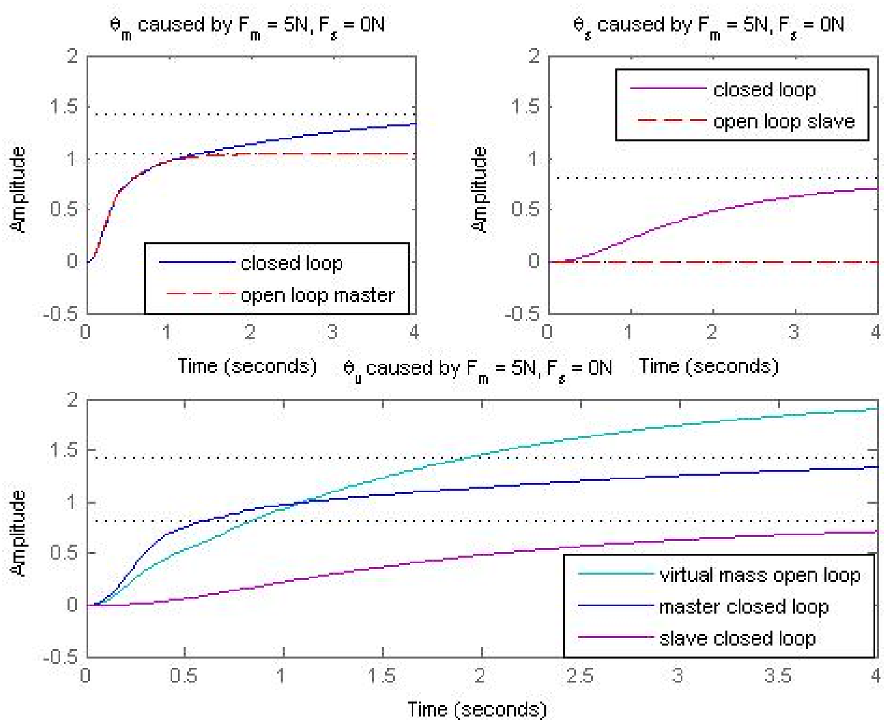


Figure 44: Criterion 2 - Second case: $kT1_{VM} < kT_m$ - $F_m = 5N$ and $F_s = 0N$ - θ_v, θ_m and θ_s

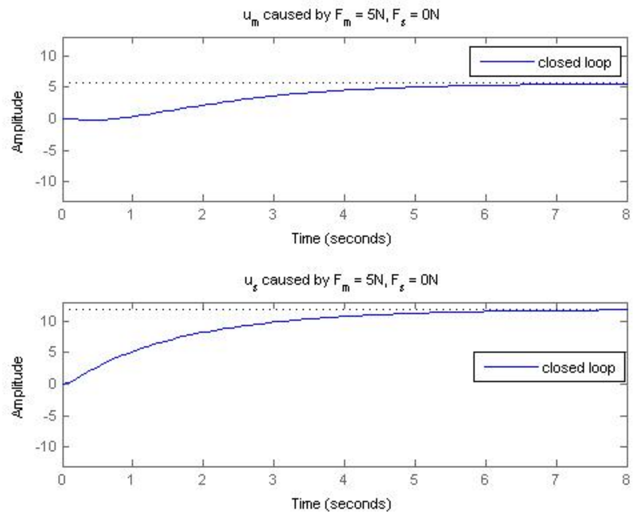


Figure 45: Criterion 2 - Second case: $kT1_{VM} < kT_m$ - $F_m = 5N$ and $F_s = 0N$ - u_m and u_s

With $F_m = 5N$ and $F_s = -2N$:

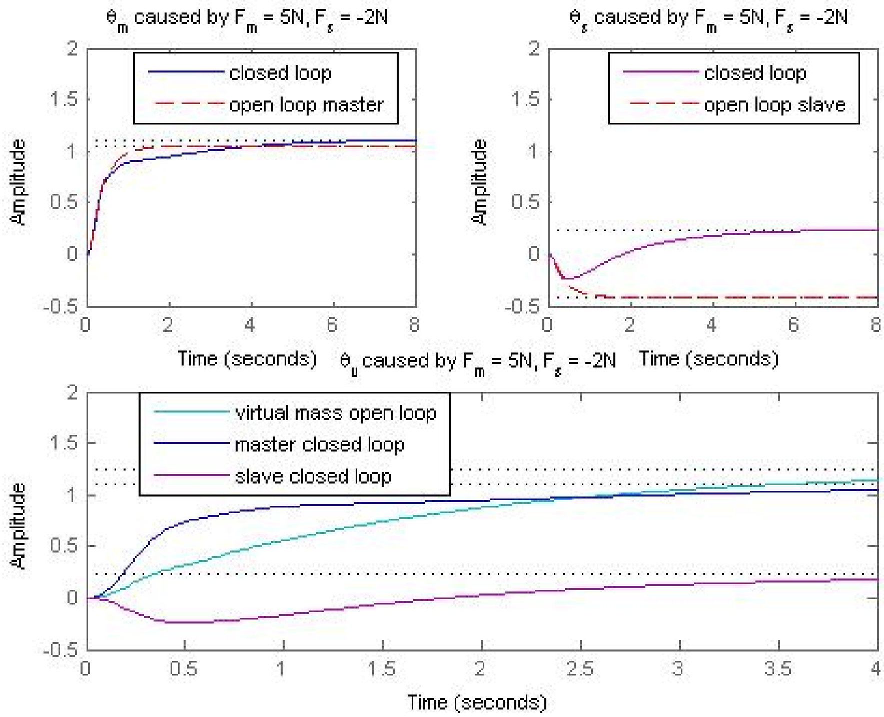


Figure 46: Criterion 2 - Second case: $kT1_{VM} < kT_m$ - $F_m = 5N$ and $F_s = -2N$ - θ_v, θ_m and θ_s

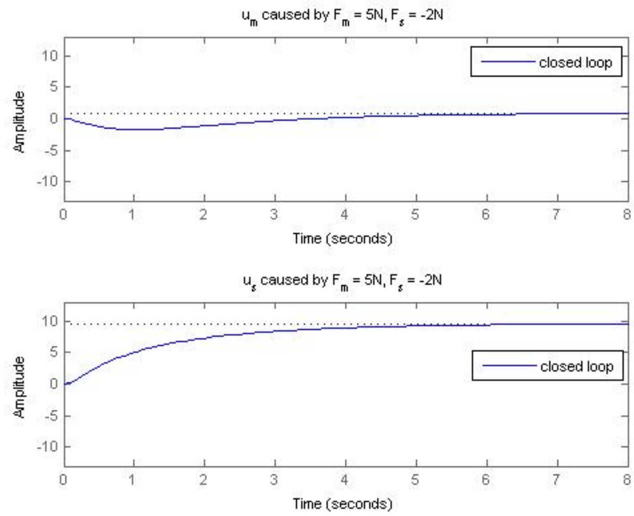


Figure 47: Criterion 2 - Second case: $kT1_{VM} < kT_m$ - $F_m = 5N$ and $F_s = 0N$ - u_m and u_s

In the above simulations, the Controller has reached some coupling - which does not seem so satisfying (shall be confirmed by the specific system's designer) between the master and the slave systems, while the Control Outputs are kept below the limit of 12V.

3.2.3 Improving results of the Second case - Working without the Control Output Limitations

Let's show that if we decide to get a bigger motor, i.e. increase the available Control Output, a better coupling between the Master and the Slave systems can be performed with these Virtual Mass system's parameters and with this criterion.

The weight functions of the Control outputs have been modified in order to achieve a better telepresence for both simulations:
$W_{\Delta\theta_{vm}} = W_{\Delta\theta_{vm}} = W_{\Delta\theta_{ms}} = (s + 300)/(s + 1)$
$W_{u_m} = W_{u_s} = 1$
$W_{n_{\theta_m}} = W_{n_{\theta_s}} = (s + 1)/(10 * (s + 1000))$

With $F_m = 5N$ and $F_s = 0N$:

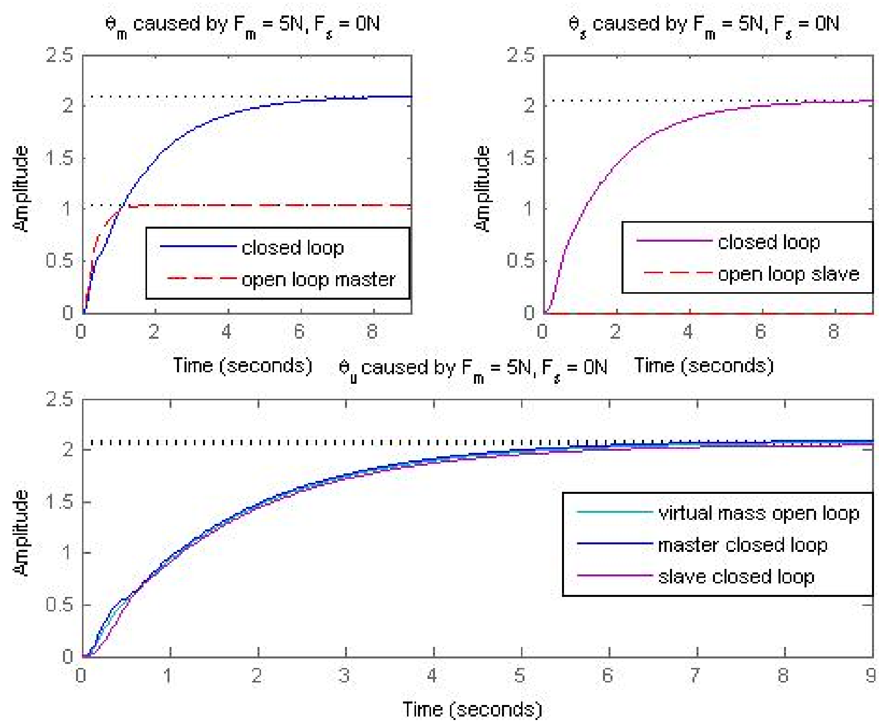


Figure 48: Criterion 2 - Second case: $kT1_{VM} < kT_m$ - $F_m = 5N$ and $F_s = 0N$ - Bigger Control Outputs - θ_v, θ_m and θ_s

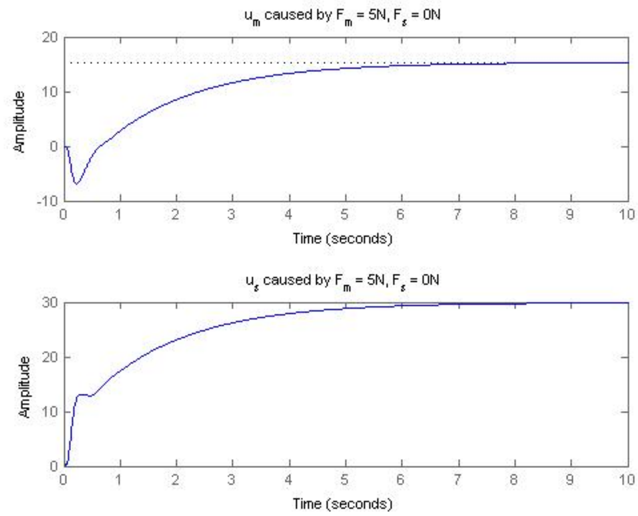


Figure 49: Criterion 2 - Second case: $kT1_{VM} < kT_m$ - $F_m = 5N$ and $F_s = 0N$ - Bigger Control Outputs - u_m and u_s

With $F_m = 5N$ and $F_s = -2N$:

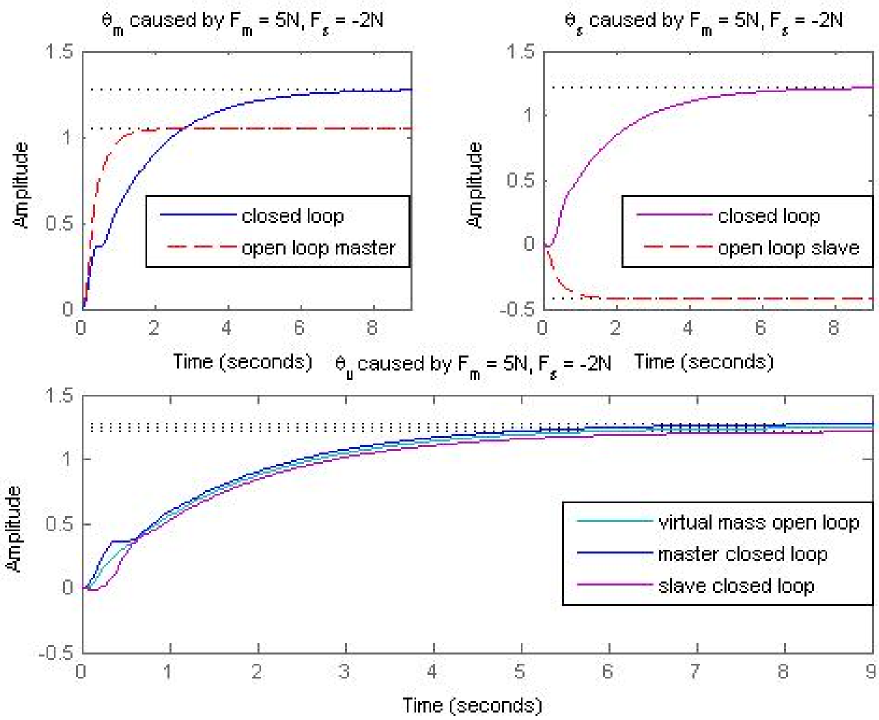


Figure 50: Criterion 2 - Second case: $kT1_{VM} < kT_m$ - $F_m = 5N$ and $F_s = -2N$ - Bigger Control Outputs - θ_v, θ_m and θ_s

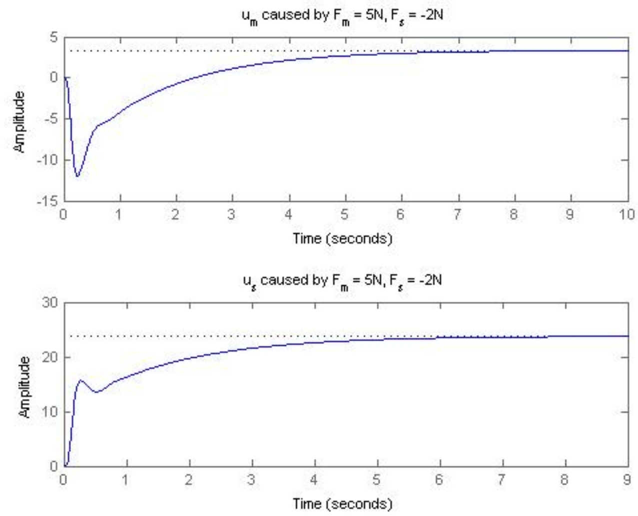


Figure 51: Criterion 2 - Second case: $kT1_{VM} < kT_m$ - $F_m = 5N$ and $F_s = 0N$ - Bigger Control Outputs - u_m and u_s

In the above simulations, the Controller has reached an almost perfect coupling between the master and the slave systems. Nevertheless, the Control Outputs had to be increased, even until 30V instead of 12V in the previous simulations.

3.2.4 Third case: Using the parameters of the Effective System found with Criterion 1

In this case, the parameters for the Virtual Mass System are those which were found in the first criterion effective system:
- k_{VM}=k_m+k_s
- $J_{VM} = 0.5 * (J_m + J_s)$
- $c_{VM} = c_m$+c_s

Using the weight functions for both simulations:
$W_{\Delta\theta_{vm}} = W_{\Delta\theta_{vm}} = W_{\Delta\theta_{ms}} = (s + 300)/(s + 1)$
$W_{u_m} = W_{u_s} = 3$
$W_{n_{\theta_m}} = W_{n_{\theta_s}} = (s + 1)/(10 * (s + 1000))$

With $F_m = 5N$ and $F_s = 0N$:

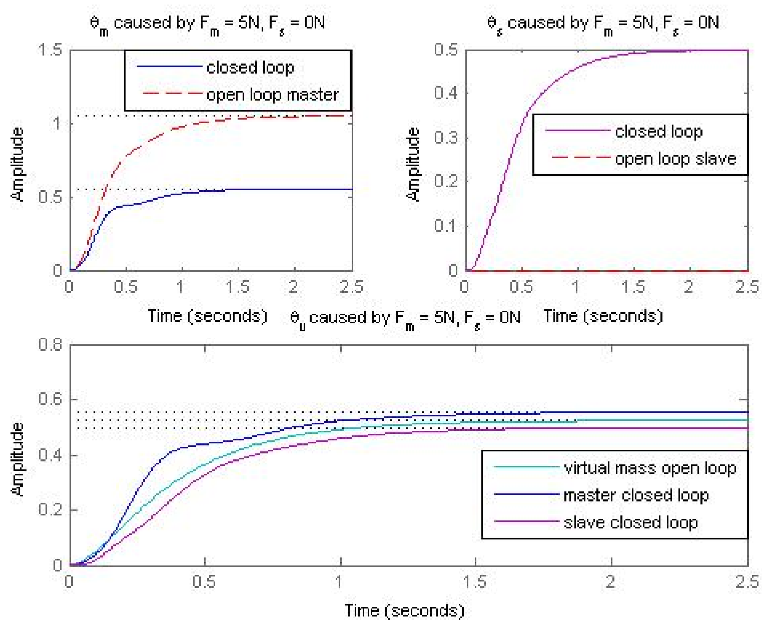


Figure 52: Criterion 2 - Third case: with Criterion 1's effective system's parameters - $F_m = 5N$ and $F_s = 0N$ - θ_v, θ_m and θ_s

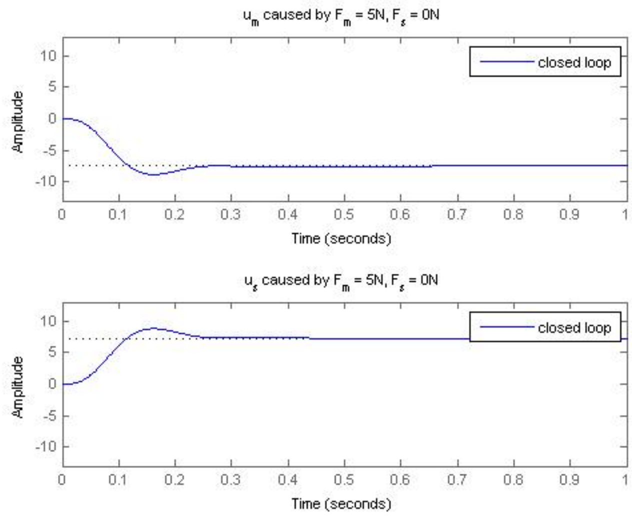


Figure 53: Criterion 2 - Third case: with Criterion 1's effective system's parameters - $F_m = 5N$ and $F_s = 0N$ - u_m and u_s

With $F_m = 5N$ and $F_s = -2N$:

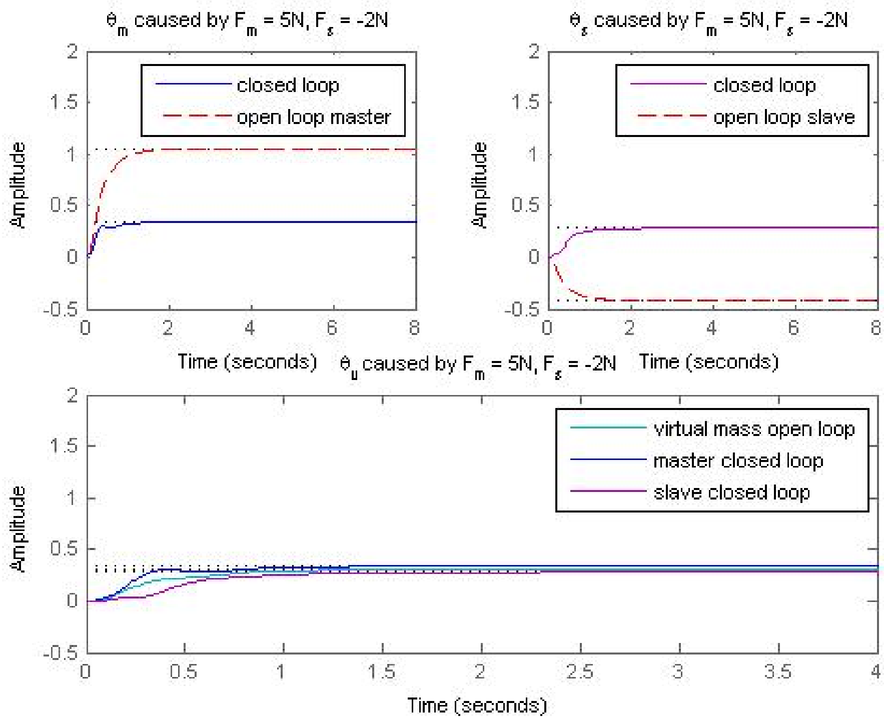


Figure 54: Criterion 2 - Third case: with Criterion 1's effective system's parameters - $F_m = 5N$ and $F_s = -2N$ - θ_v, θ_m and θ_s

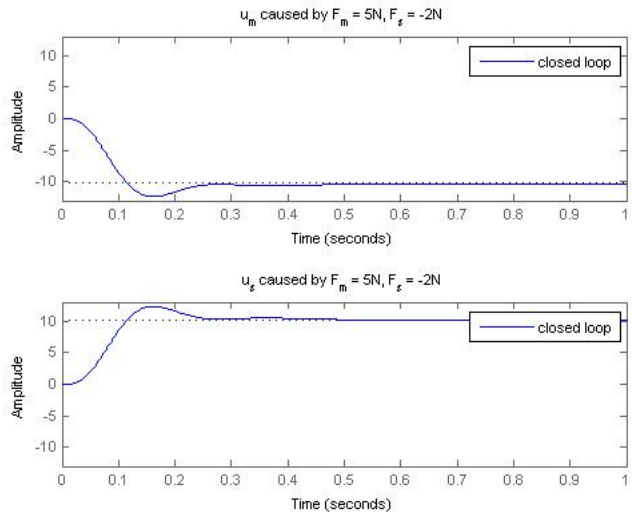


Figure 55: Criterion 2 - Third case: with Criterion 1's effective system's parameters - $F_m = 5N$ and $F_s = 0N$ - u_m and u_s

In the above simulations, the Controller has reached an almost perfect coupling between the master and the slave systems, while the Control Outputs are kept below the limit of 12V.

3.3 Third case analysis: Comparing results between Criterion 1 and Criterion 2

Recalling the relevant graphs in order to compare the simulations with the same parameters with both criterions:
With $F_m = 5N$ and $F_s = 0N$:

With Criterion 1:

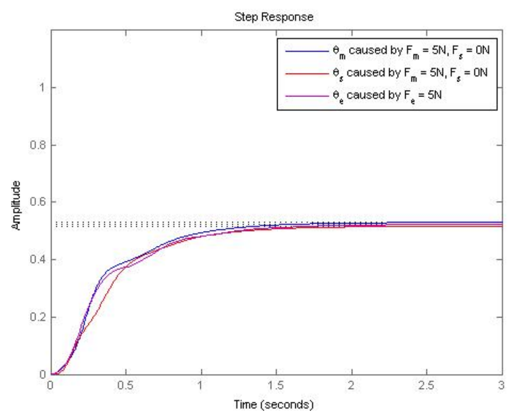


Figure 56: θ_e with $F_m = 5N$ and $F_s = 0N$

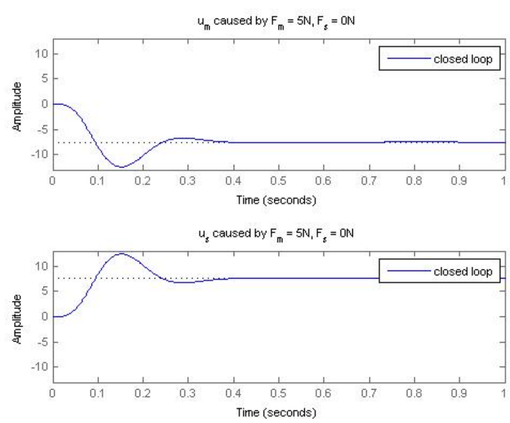


Figure 57: u_m and u_s with $F_m = 5N$ and $F_s = 0N$

With Criterion 2:

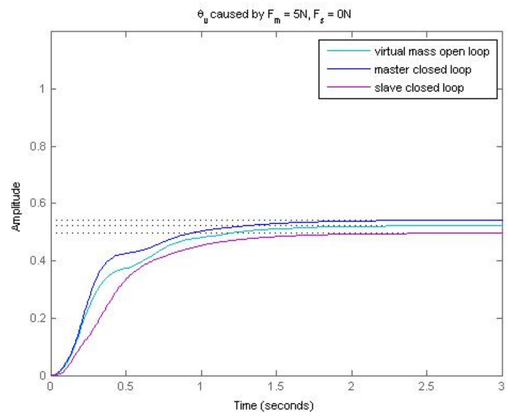


Figure 58: θ_v, θ_m and θ_s with $F_m = 5N$ and $F_s = 0N$

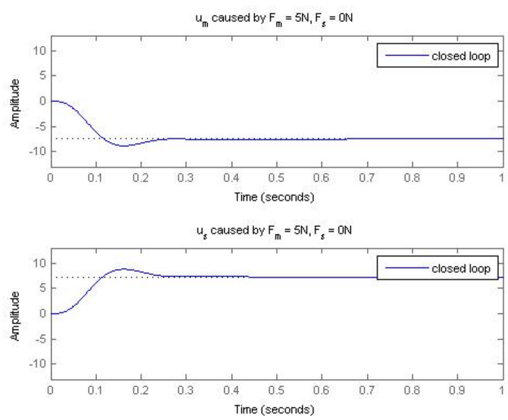


Figure 59: u_m and u_s with $F_m = 5N$ and $F_s = 0N$

The performance of the system with Criterion 1 is better than with Criterion 2. This is due to the fact that Criterion 2 is a method used to approximate the real system in an artificial way.

With $F_m = 5N$ and $F_s = -2N$:

With Criterion 1:

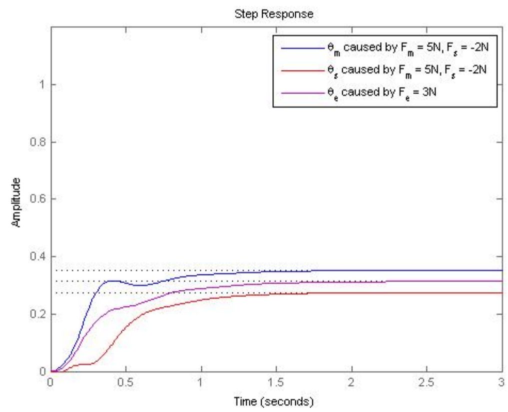


Figure 60: θ_e with $F_m = 5N$ and $F_s = -2N$

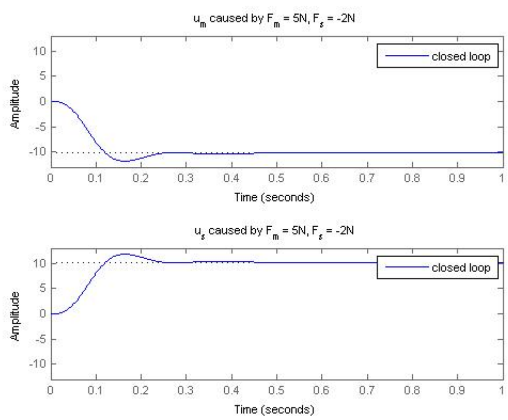


Figure 61: u_m and u_s with $F_m = 5N$ and $F_s = -2N$

With Criterion 2:

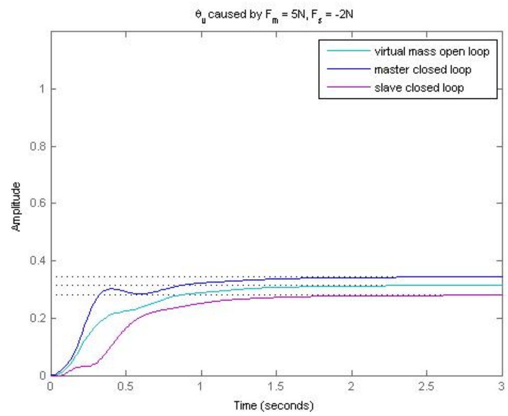


Figure 62: θ_v, θ_m and θ_s with $F_m = 5N$ and $F_s = -2N$

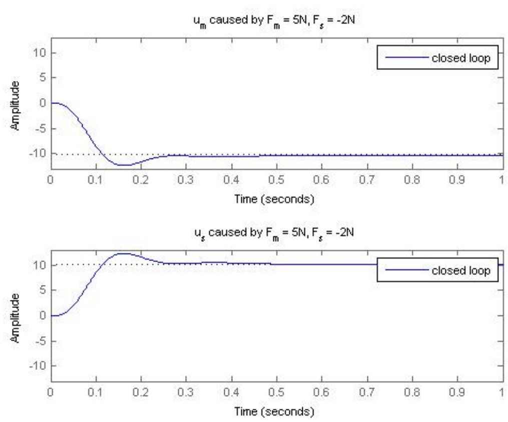


Figure 63: u_m and u_s with $F_m = 5N$ and $F_s = -2N$

In this case, the performance of the system with both criterions seem to be similar.

3.3.1 Criterion 2 - Conclusions

We have seen the below results for the different cases:

- First case: $J_{VM} > J_m$:
The Controller has reached reasonable coupling with the Control Output limitation of 12V.
- Second case: $kT_{VM} < kT_m$:
The Controller has reached reasonable coupling with the Control Output limitation of 30V.
- Third case: Setting the parameters of the Virtual Mass System equal to the effective system's parameters (found with criterion 1 analysis):
The Controller has reached reasonable coupling with the Control Output limitation of 12V.

4 Conclusion

The system has been simulated with several input forces on the Master and the Slave systems, while the synthesizing of the Optimal Controller has been performed using the following Criterions:
- Criterion 1: Coupling between the master and slave direclty
- Criterion 2: Coupling between the master and slave via a Virtual Mass System.

We have seen that both Criterions can provide optimal coupling between the Master and the Slave Systems, but few differences remain between both Criterions, and those might influence the Control Designer to choose one of these Criterions:

- With Criterion 1: Coupling between the master and slave direclty:

According to the simulations performed, the Optimal Controller reached the target: almost perfect coupling has been achieved in all cases, when the Control Outputs have been kept below 12V.
On the other hand, a limitation in the Control Design remains: looking at the graphs of θ_m and θ_s in open and closed loop, one can see that the controller makes the system very similar to the system that has been defined above and called the Effective System. As described above in this document, this Effective System is constant: its parameters are constant and cannot be modified with this Control optimization criterion.
Therefore, one can work with this criterion is order to reach telepresence, but the stiffness and damping of the closed loop system cannot be modified and this restriction might be a handicap is some cases.

- With Criterion 2: Coupling between the master and slave via a Virtual Mass System.

According to the simulations performed, the Optimal Controller reached the target: almost perfect coupling has been achieved in all cases, but the Control Outputs have not always been kept below 12V.
This Criterion gives to the Control Designer more flexibility for the system's designing: the Stiffness, the Damping and the Inertia of the system can be modified by the designer. Though this modifying of the nominal parameters may increase the required Control Output from the Motor, this flexibility can be a major advantage when such modifications are required.
The drawback of this Criterion is that the degree of the Generalized Plant required for the synthesizing of the Controller is bigger that for the first Criterion.